About an opportunity of moving of the closed mechanical system due to internal forces.

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1. *Target setting.*

For the closed system, that is the system which is not testing external influences, or in case of when the geometrical sum of external forces acting on system is equal to zero, takes place **principle of conservation of momentum.**

Thus a momentum of separate parts of system (for example, under action of internal forces) can change, but so, that the value $Q = \sum m_k v_k$ remains a constant.

Let's consider a following task (Fig. 1):

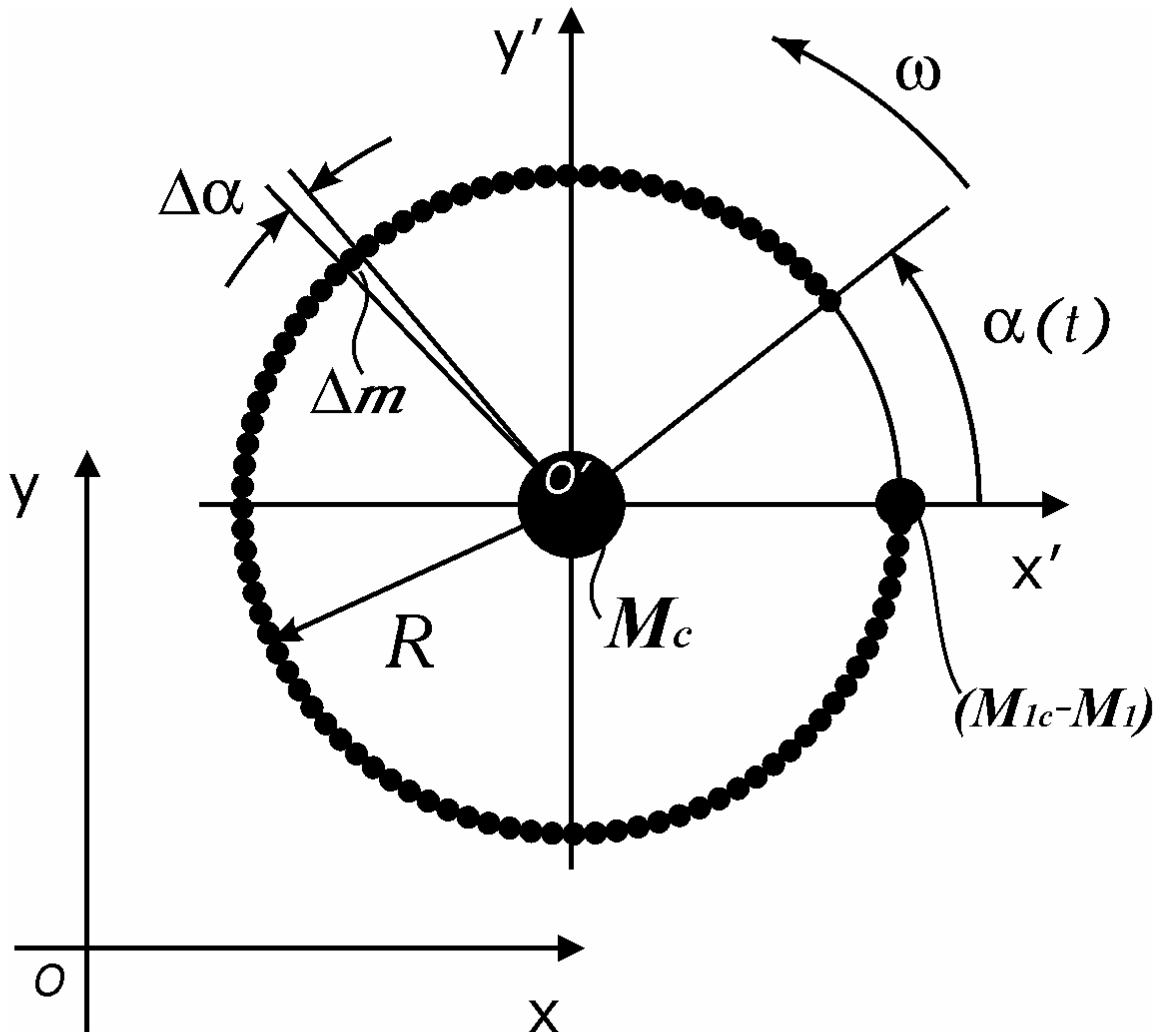

Fig. 1

Around of the center of masses of a massive body M_c, on a circle of radius $\boldsymbol{R}$ the system of bodies in total mass M_{1c} moves with constant speed. The system of bodies is allocated **continuously** and **in regular intervals**. The trajectory of movement of system of bodies is rigidly connected with a body M_c.

Moving of this system of bodies can be considered, how rotation of a body with mass M_{1c} with constant angular speed $\boldsymbol{w}$ concerning some center. We shall assume, for simplification, that each element Δm has the infinitesimal geometrical sizes.

At the certain moment of time $t_0 = 0$ "chain" of bodies is broken off. Each element of system begins stops in a point with coordinates $\left[x'_{1end} = R, y'_{1end} = 0\right]$. A stop of separate elements it is considered, how inelastic collision, after which elements get speed of a body M_c.

Other bodies of system continue to move up to a full stop, i.e., up to a angle of an aperture $\alpha(t) = 2\pi$.

As an angle of aperture $\alpha(t)$ we shall understand, in this case, an angle counted from axis O'X' up to the closing element of system body.

At the initial moment of time $(t = 0)$: $\alpha_{start} = 0$ (for the given task).

The angle of aperture changes from 0 up to 2π:

$$\alpha(t) = \int w\, dt$$

$$0 \le \alpha(t) \le 2\pi \qquad \textbf{(1)}$$

Time for which the angle of aperture varies from 0 up to 2π, we shall name " the working period ", or "running cycle" $\boldsymbol{T}$.

$0 \le \alpha(t) \le 2\pi$ at $0 \le t \le \dfrac{2\pi}{w}$

Let's consider, that all mechanical system has two degrees of freedom (moving on axes X and Y). We shall accept a condition that there is a restriction of turn of all system.

Let's construct two systems of coordinates: motionless XOY (absolute coordinates system) and mobile X'O'Y ', connected with the center of masses M_c and with the center of a trajectory of mobile elements.

2. *The analysis of movement of the closed mechanical system*

2.1. Calculation of a momentum of system of mobile elements in relative coordinates system.

Momentum of material system to equally mass of all system increased for speed of its center of inertia. In our case:

$$Q_1 = M_1\, V_{1c} \qquad \textbf{(2)}$$

Here $\boldsymbol{M}_1$ — is a mass of all elements of the system moving on a circle. From conditions of a task follows, that during the initial moment of time the mass is allocated in regular intervals. From here it is possible to accept:

$$\rho = \frac{M_{1c}}{2\pi} \tag{3}$$

ρ - angular density of distribution of elements of system of bodies on a circle (an arch of a circle).

The value $\boldsymbol{M}_1$ changes linearly in time $0 \leq \mathrm{t} \leq \dfrac{2\pi}{w}$:

$$M_1(t) = \rho\,(2\pi - \alpha\,(t)) \tag{4}$$

Whence:

$$M_1(t) = \frac{M_{1c}\,(2\pi - wt)}{2\pi} \tag{5}$$

Or, as dependence on an angle of filling of a trajectory:

$$M_1\,(\alpha) = \frac{M_{1c}\,(2\pi - \alpha(t))}{2\pi}$$

It is possible to write down also:

$$\rho = \frac{M_1(t)}{2\pi - \alpha(t)}$$

The value $(2\pi - \alpha\,(t))$ defines a angle on a trajectory filled by moving elements.

In case of our task, the center of masses of system of mobile elements in coordinates system X'O'Y' is calculated:

$$x'_{cm1} = \frac{\sum_{i=1}^{n} x'_i \, m_i}{\sum_{i=1}^{n} m_i}$$

$$y'_{cm1} = \frac{\sum_{i=1}^{n} y'_i \, m_i}{\sum_{i=1}^{n} m_i},$$

Where x_i, y_i-coordinates i a site of a body with mass m_i.

Coordinates i a mobile element in the chosen coordinates system:

$$x'_i = R\cos\big(\alpha(t)\big)$$

$$y'_i = R\sin\big(\alpha(t)\big)$$

From conditions of our task:

$$\sum_{i=1}^{n} m_i = M_1(t)$$

$$\sum_{i=1}^{n} x'_i \, m_i = \sum_{i=1}^{n} x'_i \left(\rho \, \Delta\alpha\right) = \int_{\alpha}^{2\pi} \frac{M_1(\alpha)\, R\cos\big(\alpha(t)\big)}{2\pi - \alpha(t)} \partial\alpha$$

$$\sum_{i=1}^{n} y'_i \, m_i = \sum_{i=1}^{n} y'_i \left(\rho \, \Delta\alpha\right) = \int_{\alpha}^{2\pi} \frac{M_1(\alpha)\, R\sin\big(\alpha(t)\big)}{2\pi - \alpha(t)} \partial\alpha$$

Coordinates of the center of masses of mobile elements in the chosen coordinates system are calculated by means of integrals with a variable bottom limit:

$$x'_{cm1} = \frac{1}{M_1(\alpha)} \int_{\alpha}^{2\pi} \frac{M_1(\alpha)\, R\cos\big(\alpha(t)\big)}{2\pi - \alpha(t)} \partial\alpha$$

$$y'_{cm1} = \frac{1}{M_1(\alpha)} \int_{\alpha}^{2\pi} \frac{M_1(\alpha)\, R \sin\left(\alpha(t)\right)}{2\pi - \alpha(t)} \partial\alpha \qquad \textbf{(6)}$$

After integration we shall receive:

$$\boxed{x'_{cm1} = -\frac{R \sin(\alpha(t))}{2\pi - \alpha(t)}} \qquad \textbf{(7)}$$

$$\boxed{y'_{cm1} = \frac{R\,(1 - \cos(\alpha(t)))}{2\pi - \alpha(t)}} \qquad \textbf{(8)}$$

Expressions (7)

and (8) define coordinates of the center of masses of mobile elements depending on an angle of an aperture $\alpha(t)$. From a condition of a task the angle of an aperture changes linearly eventually:

$$\alpha(t) = w\,t$$

The schedule of moving of the center of masses of system of mobile elements is presented in a Fig. 2:

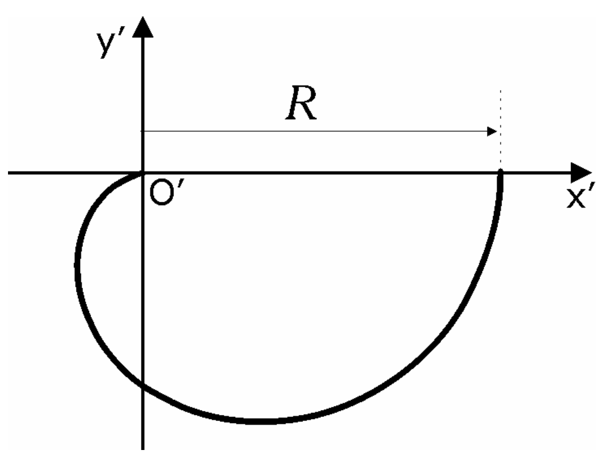

Fig. 2

In a Fig. 3 it is shown conditional moving of the center of masses of mobile system.

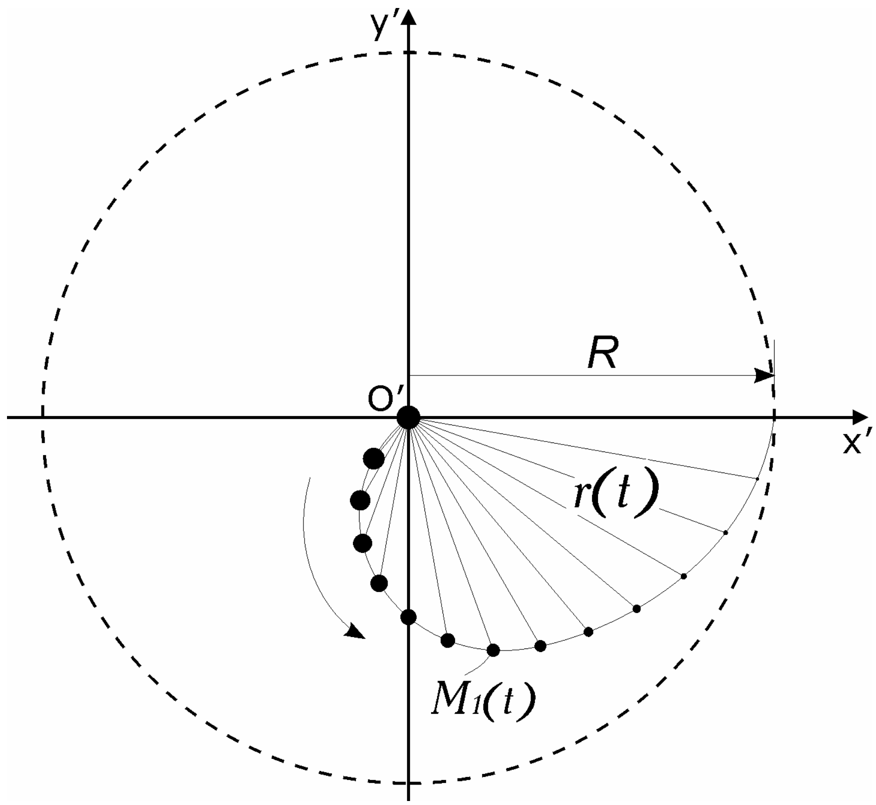

Fig. 3

Moving of the center of masses of system of mobile elements to coordinates system X'O'Y' can be compared to moving the center of masses **of a pendulum of variable length $r(t)$ and variable mass $M_1(t)$.**

In a projection to axes of coordinates system X'O'Y' momentums Q_1:

$$Q_{1x'} = \dot{x}'_{1c}\ M_1(t)$$

$$Q_{1y'} = \dot{y}'_{1c}\ M_1(t)$$

$$\dot{x}'_{1c} = \frac{d}{dt}x'_{1cm} = -\frac{R\,w\cos(\alpha)}{2\pi-\alpha} - \frac{R\,w\sin(\alpha)}{(2\pi-\alpha)^2}$$

$$\dot{y}'_{1c} = \frac{d}{dt}y'_{1cm} = -\frac{R\,w\sin(\alpha)}{2\pi-\alpha} + \frac{R\,w\,(\cos(\alpha)-1)}{(2\pi-\alpha)^2}$$

$$Q_{1x'} = M_1(t)\left(-\frac{R\,w\cos(\alpha)}{2\pi-\alpha} - \frac{R\,w\sin(\alpha)}{(2\pi-\alpha)^2}\right) \tag{9}$$

$$Q_{1y'} = M_1(t)\left(-\frac{R\,w\sin(\alpha)}{2\pi-\alpha} + \frac{R\,w\,(\cos(\alpha)-1)}{(2\pi-\alpha)^2}\right) \tag{10}$$

In a projection to axes of coordinates system XOY momentums Q_1 :

$$Q_{1x} = M_1(t)\left(\dot{x}_c - \frac{R\,w\cos(\alpha)}{2\pi-\alpha} - \frac{R\,w\sin(\alpha)}{(2\pi-\alpha)^2}\right) \tag{11}$$

$$Q_{1y} = M_1(t)\left(\dot{y}_c - \frac{R\,w\sin(\alpha)}{2\pi-\alpha} + \frac{R\,w\,(\cos(\alpha)-1)}{(2\pi-\alpha)^2}\right) \tag{12}$$

, where

$$\dot{x}_c = \frac{d}{dt}x_c(t)$$

$$\dot{y}_c = \frac{d}{dt}y_c(t)$$

Projections of speed of the beginning of coordinates of system X'O'Y' on corresponding axes of system XOY.

Let's remind: the beginning of coordinates of system X'O'Y' is connected with the center of masses body M_c

2.2. Calculation of momentum of the closed mechanical system of bodies in absolute coordinates system for the working period.

Momentum of all material system represented in a Fig. 1:

$$Q_0 = M_c\, V_c + \left(M_{1c} - M_1(t)\right) V_c + Q_1 \tag{13}$$

Here:

$\boldsymbol{Q}_0$ - a momentum of all closed system consisting of a body M_c and system of mobile elements by a lump M_{1c}

$\boldsymbol{V}_c$ - speed of the center of masses of a body M_c with the joined elements.

$\left(M_{1c} - M_1(t)\right)$ - mass of a part of the mobile elements which are not participating in movement on a circle of radius $\boldsymbol{R}$. Having stopped in a point $[x'_{1\,\mathrm{end}}=R,\quad y'_{1\,\mathrm{end}}=0]$, these elements get speed of a body M_c. (From a condition of a task.)

$\boldsymbol{Q}_1$ - a momentum of system of mobile elements. Projections $\boldsymbol{Q}_1$ on an axis of coordinates are found above (12).

As is known, the geometrical point, radius-vector r which refers to as the center of masses of material system is defined by equality

$$r_{cm} = \frac{1}{M}\sum_{k=1}^{m} m_k\, r_k \tag{14}$$

As for the working period to a body $\boldsymbol{M}_c$ additional particles join, coordinates of the center of masses of a body $\boldsymbol{M}_c$ calculate as follows:

$$x_{0c}(t) = \frac{M_c\, x_c(t) + \left(M_{1c} - M_1(t)\right)\left(x_c(t) + R\right)}{M_0} \tag{15}$$

$$y_{0c}(t)=\frac{M_c\ y_c(t)+\left(M_{1c}-M_1(t)\right)y_c(t)}{M_0}$$

Where $x_c(t)$ and $y_c(t)$ - coordinates of the center of masses of a body $\boldsymbol{M_c}$ in coordinates system XOY.

After substitution

$$M_1(t)=\frac{M_{1c}\ (2\pi-\alpha(t))}{2\pi}$$ and

$$\alpha(t)=w\,t\ ,$$

Let's receive:

$$x_{0c}(t)\ =\ \frac{2\left(M_c\ x_c\ (t)+\left(M_{1c}\ -\frac{M_{1c}\ (2\,\pi-w\,t)}{2\,\pi}\right)(x_c\ (t)+R)\right)\pi}{2\ M_c\ \pi+M_{1c}\ \ w\,t}$$

$$y_{0c}(t)\ =\ \frac{2\left(M_c\ y_c\ (t)+\left(M_{1c}\ -\frac{M_{1c}\ (2\,\pi-w\,t)}{2\,\pi}\right)y_c\ (t)\right)\pi}{2\ M_c\ \pi+M_{1c}\ \ w\,t}$$

Projections of speeds to corresponding axes of coordinates:

$$V_{0cx}=\frac{d}{dt}x_{0c}(t)$$

$$V_{0cy}=\frac{d}{dt}y_{0c}(t)$$

$$V_{0cx}\ =\ 2\left(M_c\left(\frac{d}{dt}x_c\ (t)\right)+\frac{1}{2}\frac{M_{1c}\ \ w\,(x_c\ (t)+R\ )}{\pi}\right.\quad +$$

$$+\left(M_{1c}\ -\frac{M_{1c}\ \ (2\,\pi-w\,t)}{2\,\pi}\right)\left(\frac{d}{dt}x_c\ (t)\right)\Bigg)\pi/(2\ M_c\ \pi+M_{1c}\ \ w\,t)\ -$$

$$-\ \frac{2\left(M_c\ x_c\ (t)+\left(M_{1c}\ -\frac{M_{1c}\ \ (2\,\pi-w\,t)}{2\,\pi}\right)(x_c\ (t)+R\ )\right)\pi\,M_{1c}\,w}{(2\ M_c\ \ \pi+M_{1c}\ \ w\,t)^2}$$

$$V_{0cy} = 2\left(M_c \left(\frac{d}{dt} y_c(t)\right) + \frac{1}{2}\frac{M_{1c}\ w\, y_c(t)}{\pi} \right. +$$
$$+ \left(M_{1c} - \frac{M_{1c}\ (2\pi - w\,t)}{2\pi} \right)\left(\frac{d}{dt} y_c(t)\right)\Bigg)\pi/(2\,M_c\,\pi + M_{1c}\ w\,t) -$$
$$- \frac{2\left(Mc\, y_c(t) + \left(M_{1c} - \frac{M_{1c}\ (2\pi - w\,t)}{2\pi}\right) y_c(t)\right)\pi\, M_{1c} w}{(2\,M_c\,\pi + M_{1c}\ w\,t)^2}$$

Or, after simplification:

$$\boxed{\begin{aligned} V_{0cx} &= \left(4\,M_c{}^2\left(\frac{d}{dt}x_c(t)\right)\pi^2 + 4\,M_c\left(\frac{d}{dt}x_c(t)\right)\pi\, M_{1c}\ w\,t + \right. \\ &\left. + 2\,M_{1c}\ w\,RM_c\,\pi + M_{1c}{}^2\, w^2\, t^2\left(\frac{d}{dt}x_c(t)\right)\right) \Big/ (2\,M_c\,\pi + M_{1c}\ w\,t)^2 \\ V_{0cy} &= \frac{d}{dt} y_c(t) \end{aligned}} \quad (16)$$

In a projection to axes of coordinates system XOY momentums $\boldsymbol{Q}_0$ (from (13)):

$$\begin{aligned} Q_{0x} &= M_c\, V_{0cx} + (M_{1c} - M_1(t))\, V_{0cx} + Q_{1x} \\ Q_{0y} &= M_c\, V_{0cy} + (M_{1c} - M_1(t))\, V_{0cy} + Q_{1y} \end{aligned} \quad \textbf{(17)}$$

According to the principle of conservation of momentum of the closed system:

$$\boxed{\begin{aligned} Q_{0x} &= 0 \\ Q_{0y} &= 0 \end{aligned}}$$

$$Q_{0x} = \frac{1}{2}\left(-4M_{1c}\left(-\frac{wt}{2}+\pi\right)\left(M_c\pi+\frac{M_{1c}wt}{2}\right)wR\cos(wt)+(-2M_{1c}wRM_c\pi-M_{1c}^2w^2Rt)\sin(wt)\right.$$
$$\left.+8\pi\left(-\frac{wt}{2}+\pi\right)\left(\left(M_c\pi+\frac{M_{1c}wt}{2}\right)(M_c+M_{1c})\left(\frac{d}{dt}\mathrm{x_c}(t)\right)+\frac{M_{1c}wRM_c}{2}\right)\right)/((2\pi-wt)\pi(2M_c\pi+M_{1c}wt))$$

$$Q_{0y} = \frac{-2M_{1c}\left(-\frac{wt}{2}+\pi\right)wR\sin(wt)+M_{1c}wR\cos(wt)+4\pi\left(-\frac{wt}{2}+\pi\right)(M_c+M_{1c})\left(\frac{d}{dt}\mathrm{y_c}(t)\right)-M_{1c}wR}{-2t\pi w+4\pi^2}$$

The decision of the given system of the differential equations concerning coordinates $x_c(t)$ and $y_c(t)$, in view of entry conditions:

$$x_c(0) = 0$$
$$y_c(0) = 0$$

Yields following results:

$$\mathrm{x_c}(t) = \frac{\left(-M_c\,\pi\ln(2\,M_c\,\pi+M_{1c}\,w\,t)-\frac{1}{2}M_{1c}\,\mathrm{Si}(w\,t-2\,\pi)+M_c\ln(2\,M_c\,\pi)\,\pi+\frac{1}{2}(\sin(w\,t)-\mathrm{Si}(2\,\pi))\,M_{1c}\right)R}{(M_c+M_{1c})\,\pi} \qquad \textbf{(18)}$$

$$\mathrm{y_c}(t) = \frac{1}{2}\,\frac{M_{1c}\,R\,(-\cos(w\,t)+\mathrm{Ci}(w\,t-2\,\pi)-\ln(w\,t-2\,\pi)+1-\mathrm{Ci}(-2\,\pi)+\ln(2)+\ln(\pi)+\pi\,I)}{(M_c+M_{1c})\,\pi} \qquad \textbf{(19)}$$

, where:

γ - Euler's constant:

$$\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{1}{i}\right)-\ln(n) \approx 0.5772156649...$$

Ci - Cosine integral:

$$\mathrm{Ci}(x) = \gamma+\ln(x)+\int_0^x\frac{\cos(t)-1}{t}\,dt$$

Si - Sine integral:

$$\mathrm{Si}(x) = \int_0^x \frac{\sin(t)}{t}\,dt$$

In a Fig. 4 the schedule of change of coordinates $x_c(t)$ and $y_c(t)$ for the working period is presented.

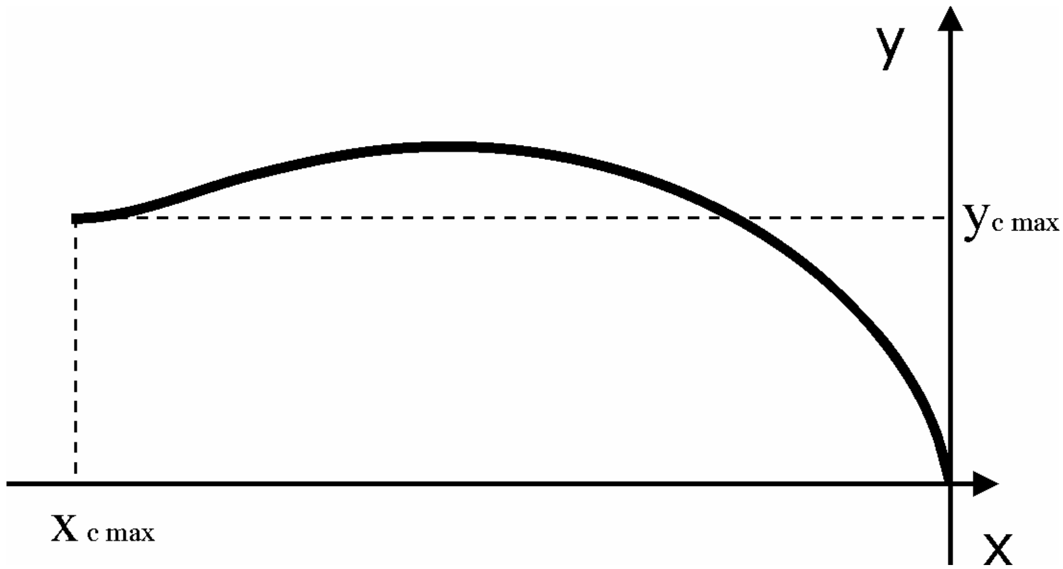

Fig. 4

Once again we shall remind, that $x_c(t)$ and $y_c(t)$ - coordinates of the center of masses of a body $\boldsymbol{M_c}$ in coordinates system XOY, that is, in "absolute" coordinates system.

$x_0(t)$ and $y_0(t)$ - coordinates of the center of masses of all closed mechanical system in coordinates system XOY.

Values $x_{c\,\max}$ and $y_{c\,\max}$ can be found as follows:

$$x_{c\max} = \lim_{t \to \frac{2\pi}{w}} x_{\mathrm{c}}(\mathrm{t})$$

$$y_{c\max} = \lim_{t \to \frac{2\pi}{w}} y_{\mathrm{c}}(\mathrm{t}) \qquad \textbf{(20)}$$

$$x_{cmax} = \frac{1}{2}\frac{R\,(-2\,M_c\,\pi\ln(M_c + M_{1c}\,) + 2\,M_c\,\pi\ln(M_c\,) - M_{1c}\,\mathrm{Si}(2\,\pi\,))}{\pi\,(M_c + M_{1c}\,)}$$

$$\boxed{y_{cmax} = \frac{1}{2}\frac{(-\mathrm{Ci}(-2\,\pi\,) + \gamma + \ln(\pi\,) + \ln(2\,)\,)\,M_{1c}\,R}{\pi\,(M_{1c} + M_c\,)}} \quad \textbf{(21)}$$

Expression

$$k = \left(-Ci(2\pi) + \gamma + \ln(2) + \ln(\pi)\right) \quad \textbf{(22)}$$

there is a value a constant.

To within 9 signs value of coefficient k makes: **(23)**

$$k = 2.437653393$$

$$y_{c\max} = \frac{k}{2\pi}\frac{M_{1c}\,R}{\left(M_c + M_{1c}\right)}$$

$$\boxed{y_{c\max} = 0.38796\frac{M_{1c}}{\left(M_c + M_{1c}\right)}R} \quad \textbf{(24)}$$

The center of masses of all system coincides with coordinate $y_c(t)$ also **(16)**.

$$y_0(t) = y_c(t)$$

Moving of the center of masses of all system of bodies looks like (Fig. 5):

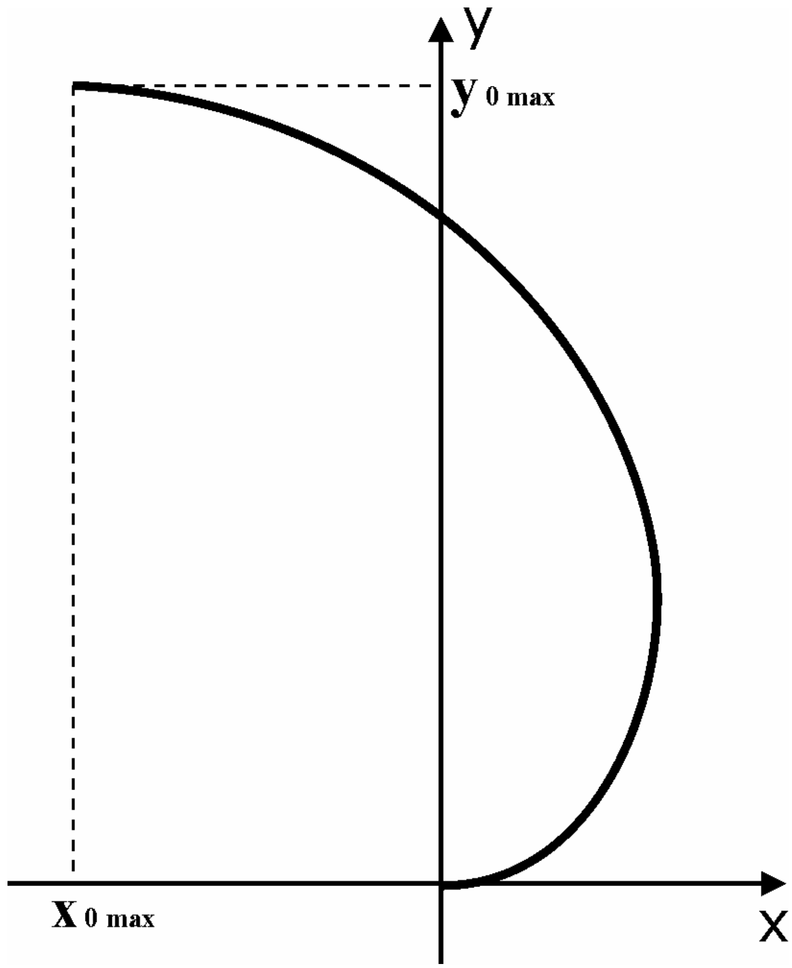

Fig. 5

$$y_{o\,\max} = y_{c\,\max}$$

$$\mathrm{y}_0(t) = \frac{1}{2}\,\frac{(\pi\, I + \mathrm{Ci}(w\,t - 2\,\pi) - \ln(w\,t - 2\,\pi) - \mathrm{Ci}(-2\,\pi) + \ln(\pi) + \ln(2))\,R\,M_{1c}}{\pi\,(M_{1c} + M_c)} \tag{25}$$

$$x_0(t) = -\frac{1}{2}\,\frac{(2\,M_c\,\pi\,\ln(M_{1c} + M_c) - 2\,M_c\,\pi\,\ln(M_c) + (-2\,\pi + \mathrm{Si}(2\,\pi))\,M_{1c}\,)\,R}{\pi\,(M_{1c} + M_c)} \tag{26}$$

$$\boxed{y_{o\,\max} = \lim_{t\to\frac{2\pi}{w}} y_0(\mathrm{t}) = y_{c\,\max}}$$

$$x_{o\,\max} = \lim_{t \to \frac{2\pi}{w}} x_0(\mathrm{t}) = \frac{\left(M_c\, \pi \ln(M_{1c} + M_c) - M_c\, \pi \ln(M_c) + \left(-\pi + \frac{1}{2}\,\mathrm{Si}(2\,\pi) \right) M_{1c} \right) R}{\pi\,(M_{1c} + M_c)}$$

Dependence of moving of the center of masses of all closed system are presented in a Fig. 6 and a Fig. 7.

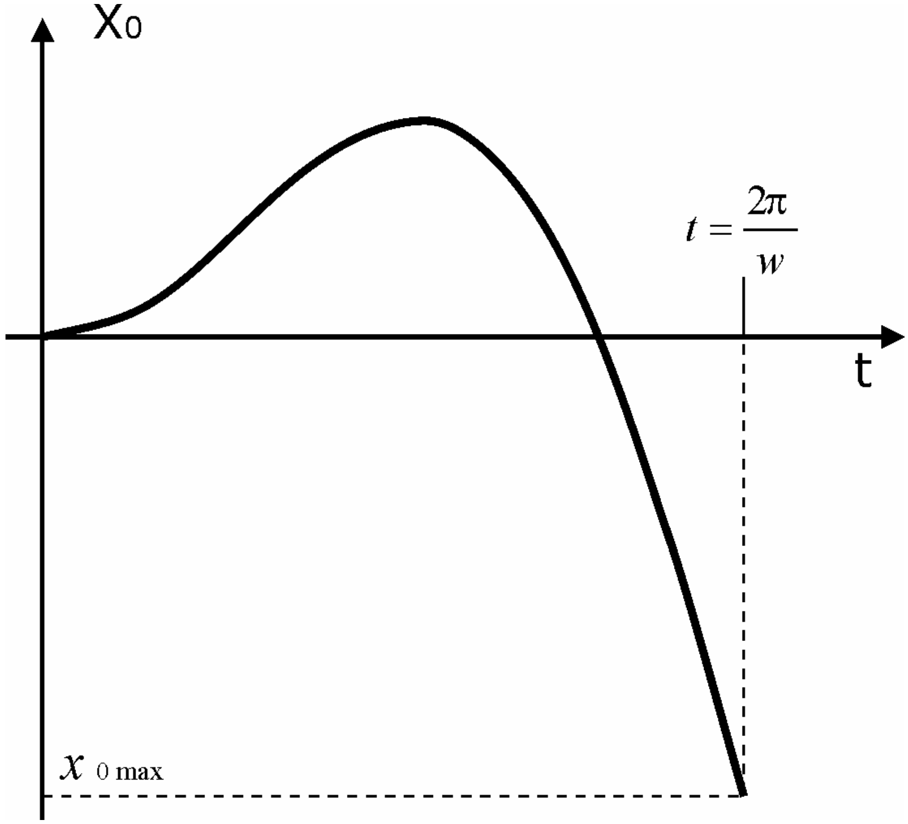

Fig. 6

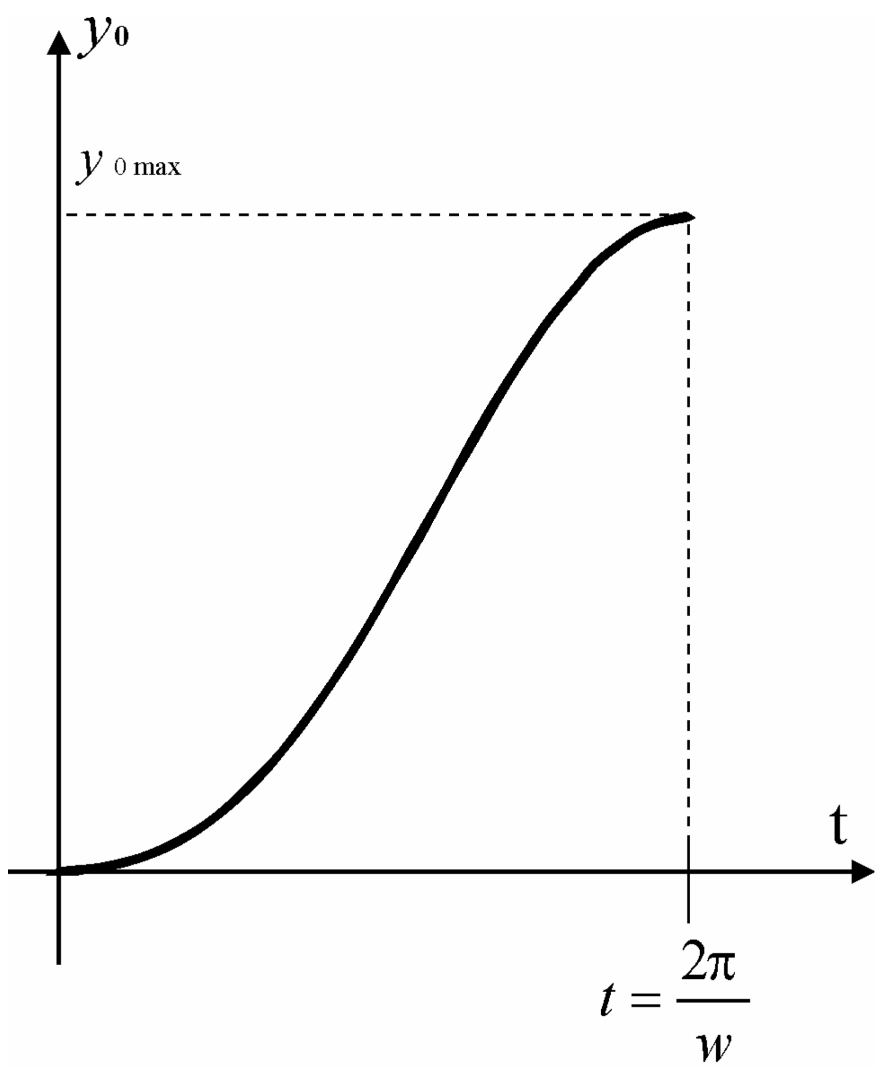

Fig. 7

Conclusions.

Under condition of preservation of a momentum the given system (Fig. 1) moves for the certain time interval on the certain distance.

3. *The analysis of movement of mechanical system (Fig. 1) by means of Lagrange's equations.*

Lagrange's equations for holonomic systems generally look like:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i \quad (i = 1, 2.., n)$$

Where q_i - the generalized coordinates, which number it is equal to number **n** degrees of freedom of system, $\dot{q}_i$ - the generalized speeds, Q_i - the generalized forces, T - the kinetic energy of system expressed through q_i and $\dot{q}_i$.

For the system represented in a Fig. 1, for the generalized coordinates it is possible to accept coordinates of the center of masses of a body M_c: $x_c(t)$ and $y_c(t)$

Kinetic energy of system develops of kinetic energy of a body M_c which for the working period particles join, and kinetic energy of system of mobile elements of variable mass $M_1(t)$.

$$T = T_0 + T_1 \tag{27}$$

$$T_0 = \frac{M_0\left(V^2{}_{cx} + V^2{}_{cy}\right)}{2}, \text{ where} \tag{28}$$

$$M_0 = M_c + \left(M_{1c} - M_1(t)\right)$$

$$V_{cx} = \frac{d}{dt}x_c(t)$$

$$V_{cy} = \frac{d}{dt}y_c(t)$$

Kinetic energy of system of mobile elements develops of kinetic energy of progress of the center of masses and rotary concerning the center of masses.

$$T_1 = \frac{M_1(t)\,V^2{}_1}{2} + \frac{I_c w^2}{2}$$

$$T_1 = \frac{M_{1c}\ (2\pi - w\,t)\left((V_{cx} + V_{x1})^2 + (V_{cy} + V_{y1})^2\right)}{4\pi} + \frac{I_c\ w^2}{2}$$

Speed of the center of masses of system of mobile elements develops of own speed V_1 and portable V_c.

$(V_{cx}+V_{x1})$ and $(V_{cy}+V_{y1})$ - according to a projection of speed of the center of masses of system of mobile elements to axes OX and OY.

I_c - the moment of inertia of system of mobile elements concerning the center of masses of system.

$$T=\frac{M_0\,(V_{0x}{}^2+V_{0y}{}^2)}{2}+\frac{M_{1c}\,(2\,\pi-w\,t)\,((V_{cx}+V_{x1})^2+(V_{cy}+V_{y1})^2)}{4\,\pi}+\frac{I_c\,w^2}{2}$$

$$\frac{\partial}{\partial V_{cx}}T=M_0\,V_{0x}+\frac{M_{1c}\,(2\,\pi-w\,t)\,(2\,V_{cx}+2\,V_{x1})}{4\,\pi}$$

$$\frac{\partial}{\partial V_{cy}}T=M_0\,V_{0y}+\frac{M_{1c}\,(2\,\pi-w\,t)\,(2\,V_{cy}+2\,V_{y1})}{4\,\pi}$$

$$\frac{d}{dt}\left(\frac{\partial}{\partial V_{cx}}T\right)=\frac{1}{8}\left(32\,\pi^2\left(-\frac{w\,t}{2}+\pi\right)^2(M_c+M_{1c})\left(\frac{d^2}{dt^2}\mathrm{x_c}(t)\right)+16\,R\times\right.$$
$$\times\,w^2\left(\left(\frac{1}{4}w^2\,\pi\,t^2-w\,\pi^2\,t-\frac{1}{4}\pi+\pi^3\right)\sin(w\,t)-\right.$$
$$\left.\left.-\frac{1}{2}\left(-\frac{w\,t}{2}+\pi\right)\left(\frac{\pi}{2}+\pi\cos(w\,t)-\frac{w\,t}{4}\right)\right)M_{1c}\right)\Big/\left((2\,\pi-w\,t)^2\,\pi^2\right)$$

$$\frac{d}{dt}\left(\frac{\partial}{\partial V_{cy}}T\right) = \frac{1}{2}\left(2\,\pi\,(w\,t-2\,\pi)^2\,(M_c+M_{1c})\left(\frac{d^2}{dt^2}\mathrm{y_c}(t)\right)+M_{1c}\;w^2\times\right.$$

$$\left.\times\;((4\,\pi\,w\,t-w^2\,t^2-4\,\pi^2+1)\cos(w\,t)-1+(w\,t-2\,\pi)\sin(w\,t))\,R\right)\Big/$$

$$\Big/(\pi\,(w\,t-2\,\pi)^2)$$

$$\frac{\partial T}{\partial x_c}=0$$

$$\frac{\partial T}{\partial y_c}=0$$

On a condition of a task, on our mechanical system external forces don't operate. Therefore:

$$\frac{d}{dt}\left(\frac{\partial}{\partial V_{cx}}T\right)+\frac{\partial T}{\partial x_c}=0$$

$$\frac{d}{dt}\left(\frac{\partial}{\partial V_{cy}}T\right)+\frac{\partial T}{\partial y_c}=0 \qquad \textbf{(29)}$$

The decision of system of the differential equations of the second order with respect to $x_c(t)$ and $y_c(t)$, in view of entry conditions:

$$x_c(0)=0$$

$$y_c(0)=0$$

$$\frac{d}{dt}x_c(0)=0$$

$$\frac{d}{dt}y_c(0)=0$$

yields following results:

$$x_c(t) = \frac{1}{2}\frac{M_{1c}\ R\,(\sin(w\,t) - \mathrm{Si}(w\,t - 2\,\pi) - w\,t - \mathrm{Si}(2\,\pi))}{\pi\,(M_c + M_{1c})} \tag{30}$$

$$y_c(t) = -\frac{1}{2}\frac{M_{1c}\ R\,(\ln(w\,t - 2\,\pi) + \cos(w\,t) - \mathrm{Ci}(w\,t - 2\,\pi) - \ln(2) - \ln(\pi) - 1 + \mathrm{Ci}(-2\,\pi))}{\pi\,(M_c + M_{1c})} \tag{31}$$

Value of coordinate y_c during the moment of time $t = \frac{2\pi}{w}$:

$$y_{cmax} = \frac{1}{2}\frac{(\gamma + \ln(2) - \mathrm{Ci}(-2\,\pi) + \ln(\pi))\,M_{1c}\ R}{\pi\,(M_c + M_{1c})} \tag{32}$$

Or, approximately, to within 5 signs:

$$y_{cmax} = \frac{0.38797\,M_{1c}\ R}{M_c + M_{1c}}$$

That in accuracy corresponds to the results received earlier (21) and (23).

Value of coordinate x_c during the moment of time $t = \frac{2\pi}{w}$:

$$x_{cmax} = \frac{-M_{1c}\ R\ \mathrm{Si}(2\,\pi) - 2\,M_{1c}\ R\ \pi}{2\,M_c\,\pi + 2\,\pi\,M_{1c}} \tag{33}$$

$$x_{cmax} \approx -\frac{1.2257\,M_{1c}\ R}{M_c + M_{1c}}$$

Comparison of expressions (33) and (20) shows, that at $M_{1c} \ll M_c$,

values $x_{c\max}$ are practically equal. (at $M_c = 10\,M_{1c}$ the error makes $< 0.5\%$. Probably, it is connected with a method of the decision of the differential equations.).

The analysis of movement of mechanical system by means of Lagrange's equations also shows an opportunity of moving of the closed mechanical system without external influence.

4. *Conclusions*

Calculation of moving of the center of masses of the closed mechanical system is executed on the basis of the law of preservation of a momentum and by means of Lagrange's equations.

In time T the total momentum of system is equal to zero (Fig. 8). Moving of all closed system arises only <u>during the working period</u>.

In a Fig. 8 drawings of change of momentums for the working period are presented:

systems of mobile elements-

$$M_1(t) \left(V_{1y} + V_{cy}\right)$$

and "motionless", cases-

$$\left(M_c + M_{1c} - M_1(t)\right) V_{cy}$$

The drawing of a total momentum, that is, quantities of movement of all system, coincides with an axis of abscissas of the drawing.

If at any moment to stop system of mobile elements, all mechanical system will have initial speed.

If during the initial moment of time all mechanical system had zero speed in system XOY after a stop of system of mobile elements speed of all mechanical system will be equaled also <u>to zero</u>!

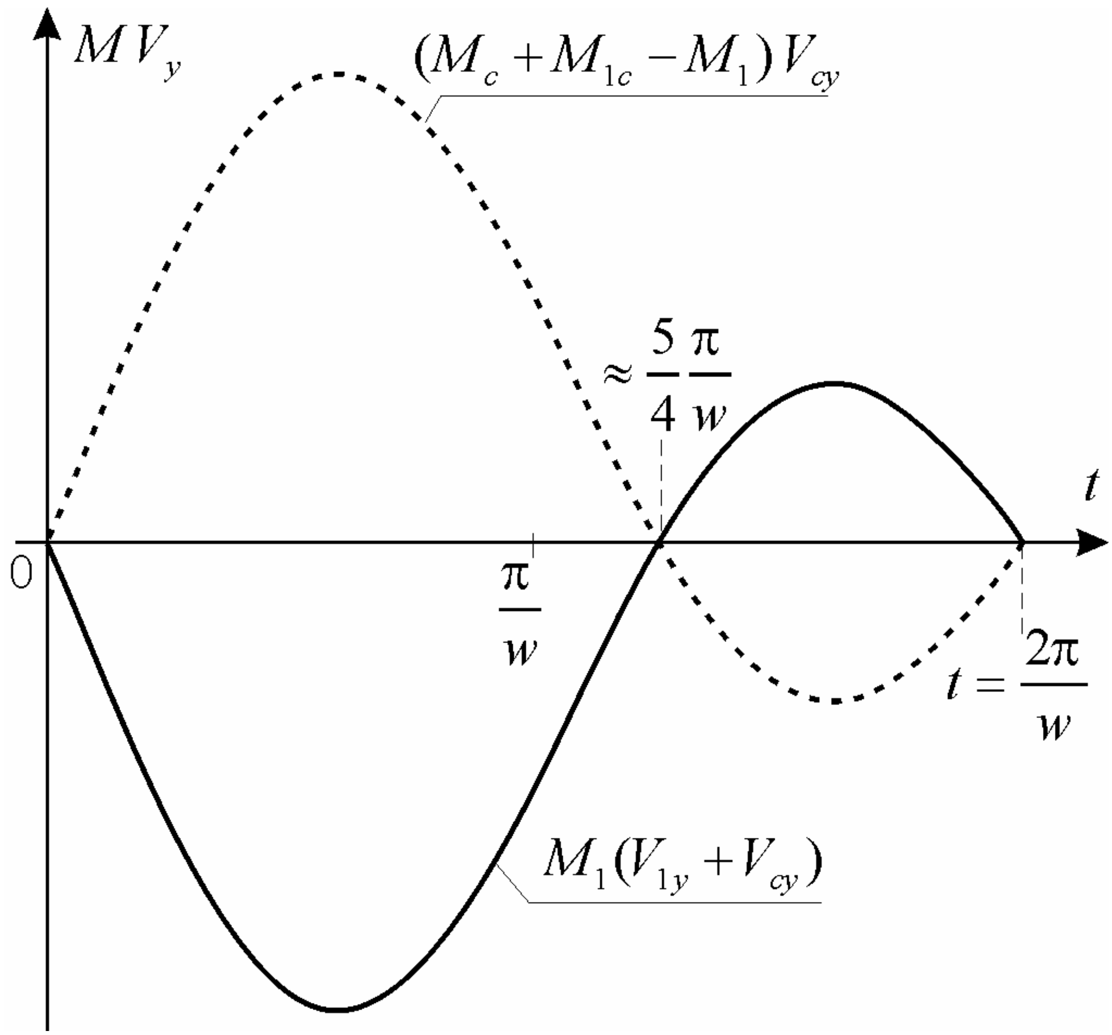

Fig. 8

It is possible to speak only about conditional speed and conditional acceleration of the center of masses of all mechanical system for the working period.

Speed of system is a derivative on time from coordinate.

Let's leave while calculation on one of coordinates. Below it will be explained; in this connection it is connected.

The greatest interest represents change of coordinate $y_0(t)$ (25) or (31).

$$V_y = \frac{d}{dt} y_0(t)$$

$$V_y = \frac{1}{2}\frac{w\,(\cos(w\,t) - 1)\,M_{1c}\ R}{(w\,t - 2\,\pi)\,(M_c + M_{1c}\ )\,\pi} \tag{34}$$

$$a_y = \frac{d}{dt}V_y$$

$$a_y = -\frac{1}{2}\frac{M_{1c}\ w^2\,R\,(\sin(w\,t)\,w\,t - 2\sin(w\,t)\,\pi + \cos(w\,t) - 1)}{(w\,t - 2\,\pi)^2\ (M_c + M_{1c}\ )\,\pi} \tag{35}$$

Corresponding drawings of change of speed and acceleration of all closed mechanical system are presented in a Fig. 9 and a Fig. 10

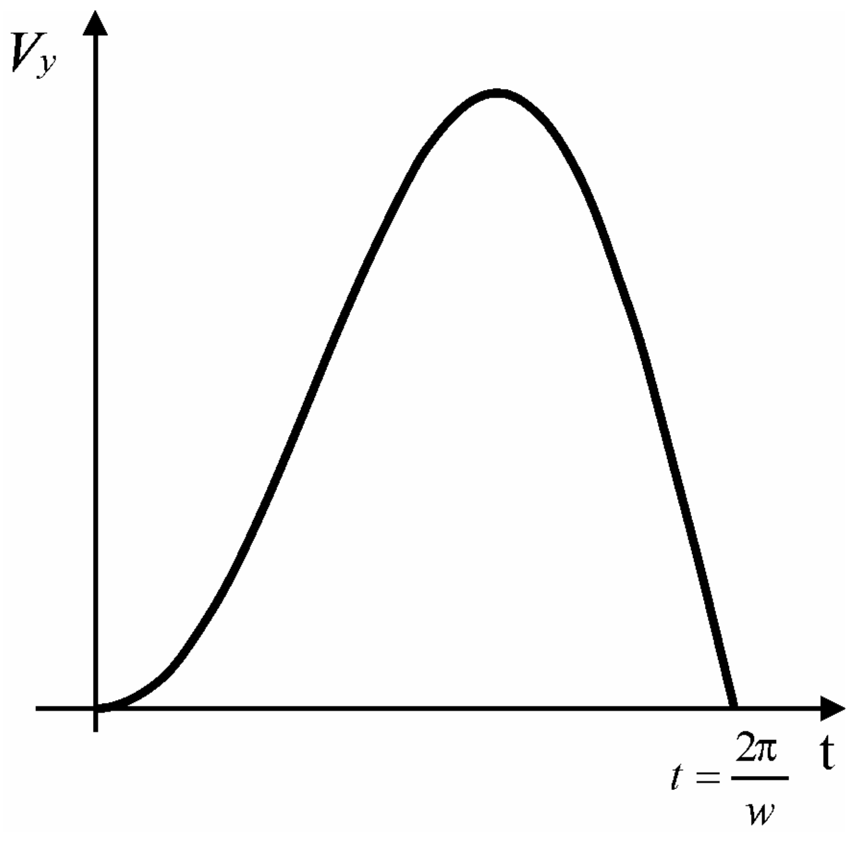

Fig. 9

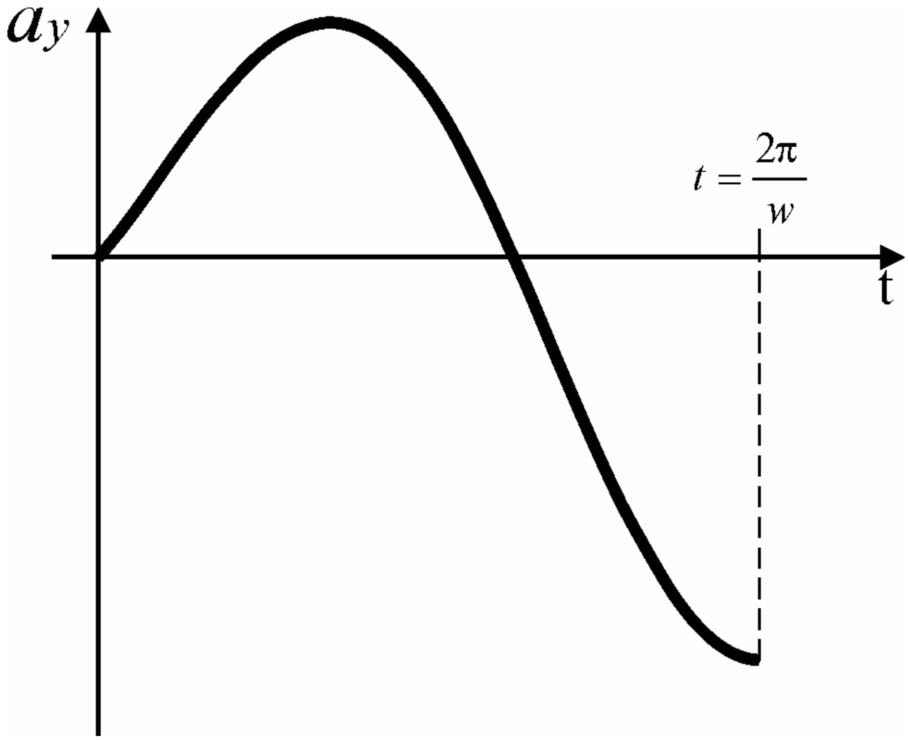

Fig. 10

4.1. Calculation of the force developed by system of mobile elements.

According to the same theorem, change of momentum of system of bodies to equally sum of all external forces enclosed to system body.

$$\frac{dQ}{dt} = \sum F_k^e$$

On conditions of our task this equality is carried out: external forces are absent also change of momentum of all system to equally zero.

At the same time, the schedule on a Fig. 5 describing moving of system of bodies isn't rectilinear.

Let's try to find force which could cause similar moving for the same interval of force for a body, the same mass, as well as mass of all closed system.

Force $\vec{F}$ we shall define in projections to axes: F_x and F_y

Corresponding projections of acceleration which would be caused by the given force, are defined, how:

$$a_{0x} = \frac{F_x}{(M_c + M_{1c})}$$

$$a_{0y} = \frac{F_y}{\left(M_c + M_{1c}\right)}$$

Let's make system of the equations:

$$\begin{aligned} x_0\left(t\right) &= \iint a_{0x}\ dtdt \\ y_0\left(t\right) &= \iint a_{0y}\ dtdt \end{aligned} \qquad \textbf{(36)}$$

Where: $x_0\left(t\right)$ and $y_0\left(t\right)$ coordinates of position of the center of masses of all closed system ((25) and (26));

The decision of system of the equations (36) results in the following:

$$\begin{aligned} F_x = {} & R(2\,M_c\,\pi\,(-\ln(2\,M_c\,\pi + M_{1c}\ \ w\,t) + \ln(\pi) + \ln(M_c\,) + \ln(2\,)) + \\ & + M_{1c}\ \ (w\,t - \mathrm{Si}(2\,\pi) - \mathrm{Si}(w\,t - 2\,\pi)))/(t^2\,\pi) \end{aligned} \qquad \textbf{(37)}$$

$$F_y = \frac{R(\ln(2\,) + \ln(\pi) + \pi\,I + \mathrm{Ci}(w\,t - 2\,\pi) - \ln(w\,t - 2\,\pi) - \mathrm{Ci}(-2\,\pi))\,M_{1c}}{t^2\,\pi} \qquad \textbf{(38)}$$

Average values of functions F_x and F_y for the period $0 < t < \frac{2\pi}{w}$:

$$F_{xcp} = \frac{1}{4}\frac{Si\left(2\pi\right)M_{1c}\ w^2 R}{\pi^3}$$

$$F_{ycp} = \frac{1}{4}\frac{\left(\pi I + \ln\left(\pi\right) + \ln\left(2\right) - Ci\left(-2\pi\right) + \gamma\right)M_{1c}\ w^2 R}{\pi^3}$$

Force of moving is result of centripetal forces of inertia.

This conclusion can be made on the basis of proportionality of values F_x and F_y to expression : $w^2 R$.

Plots of function F_x and F_y are presented in a Fig. 11 and a Fig. 12:

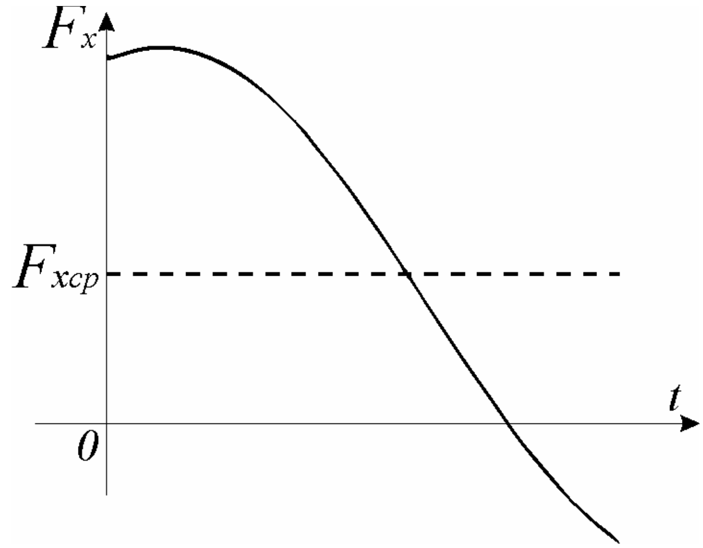

Fig. 11

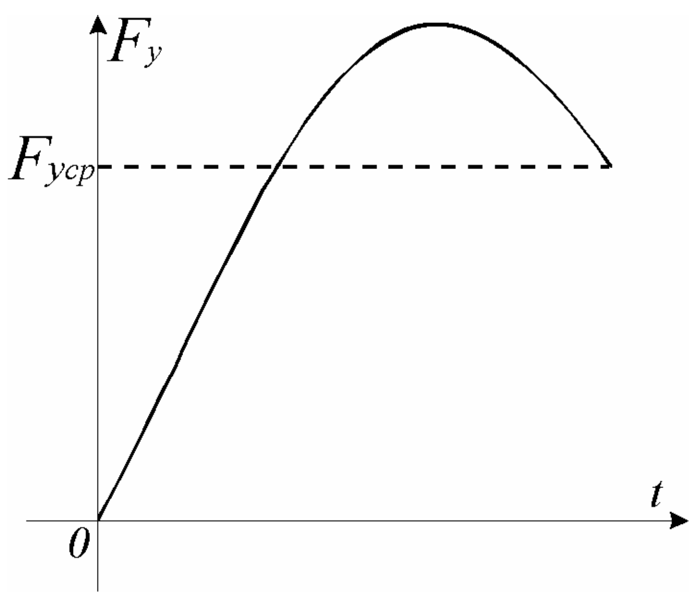

Fig. 12

4.2. Work of force and power.

At movement of mechanical system the sum of works of all operating forces on some moving is equal to change of its kinetic energy T, that is:

$$\sum A_i = T_1 - T_0$$

Where T_1 and T_0 - value of kinetic energy in initial and final positions of system.

For our considered task:

$$T_1 = T_0 = 0$$

that is, kinetic energy of all system in initial and final positions is equal to zero.

Hence, the force developed by system of mobile elements, doesn't make work.

The system of mobile elements for the working period changes the kinetic energy in an interval:

$$0 \le T \le \frac{1}{2} M_{1c} w^2 R^2 \tag{39}$$

Increase of kinetic energy is a translation of system of mobile elements in a working condition.

Decrease of kinetic energy - a stop of mobile elements during a running cycle.

From this interval we shall define work on moving mobile elements:

$$|A| = \frac{1}{2} M_{1c} w^2 R^2 \tag{40}$$

The power developed by system of mobile elements for the working period:

$$N = \frac{A}{t} = \frac{A w}{2\pi} = \frac{1}{4\pi} M_{1c} w^3 R^2 \tag{41}$$

The note:

Values of work and capacity are found from a condition, that elements of mobile system completely stop after end of a running cycle. In real devices it to do it is inexpedient. It is desirable to change speed of mobile elements only on one coordinate.

In that case, it is possible to use a stock of kinetic energy of mobile system for translation of FE in a working condition. Thus values (40) and (41) will be much less.

5. ***Group* work. Association of several systems of mobile elements.**

Process of moving of the considered system $y_0(t)$ can be repeated.

For this purpose it is required to move mass M_{1c} which gathers on distance R from the center of masses M_c, to the center of masses M_c. **Not breaking position of the center of masses** of all system, in regular intervals to disperse system of mobile elements M_{1c} on a circle of radius R. To give to system of mobile elements angular speed W. After that, it is possible to repeat a running cycle (a Fig. 25, a Fig. 26).

To exclude from calculations coordinate $x_0(t)$ and $\Omega_0(t)$ (where $\Omega_0(t)$ - a angle of turn of all mechanical system), it is convenient to bring in the closed mechanical system one more system of the mobile elements making movement, mirror symmetric to the first system of mobile elements concerning an axis $O'Y'$.

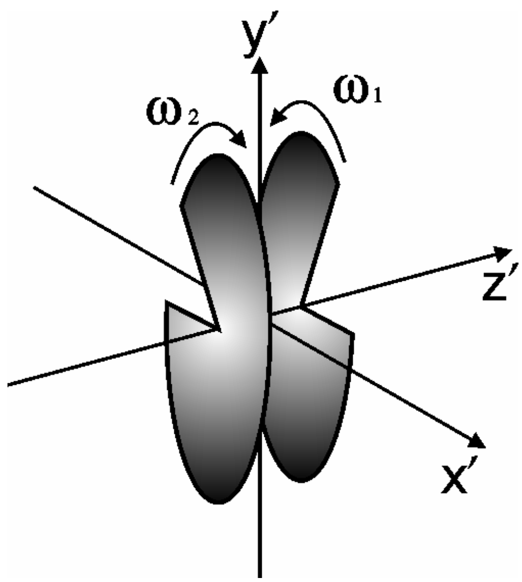

Fig. 13

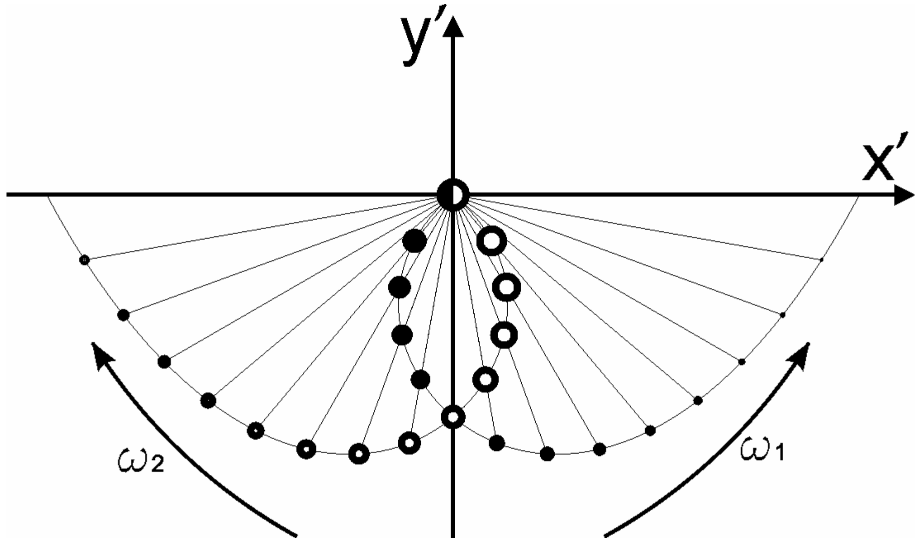

Fig. 14

It is meant mirror symmetry of movement:

Starting angle: $\alpha_{2\,start} = \pi - \alpha_{1\,start}$

Angular speed: $w_2 = -w_1$

Change of mass of mobile elements: $M_2(t) = M_1(t)$

Elements of mobile system stop in a point with coordinates $\left[x'_{2\,end} = -R, y'_{2\,end} = 0\right]$

Coordinates of the center of masses of the second mobile system

$$x_{cm2}(t) = \frac{R\sin(wt)}{2\pi - wt} = -x_{cm\,1}(t)$$

$$y_{cm2}(t) = \frac{R\,(1-\cos(wt))}{2\pi - wt} = y_{cm\,1}(t)$$

In a projection to axes of coordinates system X'O'Y' a momentum Q_2:

$$Q_{2x} = M_2\,\frac{d}{dt}x_{cm2} = -Q_{1x}$$

$$Q_{2y} = M_2\,\frac{d}{dt}y_{cm2} = Q_{1y}$$

The total momentum of systems of mobile elements X' is equal a projection to an axis to zero.

$$Q_{1x} + Q_{2x} = 0$$

The total momentum of two systems of mobile elements Y' is equal a projection to an axis to the double momentum of one of systems of mobile elements.

$$Q_{1y} + Q_{2y} = 2\,Q_{1y}$$

The sum of the moments of momentums of two systems of mobile elements is equal to zero

$$K_{1z} + K_{2z} = 0$$

As it was already mentioned above (Fig. 3), to coordinates system X'O'Y' it is possible to compare moving of the center of masses of system of mobile elements to moving the center of masses **of a pendulum of variable length $r(t)$ and variable mass $M_{12}(t)$.** For a case of two systems of mobile elements, moving of their general center of masses can be presented as back and forth motion of a body of variable mass on one coordinate. (Fig. 15).

That is, the mechanical system consisting of two systems of mobile elements possesses one degree of freedom.

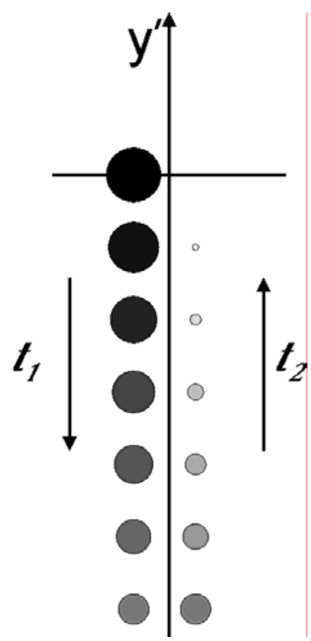

Fig. 15

*

During the first half-cycle $0 < t_1 <\approx \frac{5}{4}\frac{\pi}{w}$ there is a reduction of mass of two systems of mobile elements. Thus coordinate y'_{12} (the center of masses of systems of mobile elements) - decreases.

During the second half-cycle $\frac{5}{4}\frac{\pi}{w} \approx< t_2 < \frac{2\pi}{w}$, the coordinate y'_{12} increases, but the mass M_{12} continues to decrease up to zero value. $t_1 \neq t_2$

6. *Organization of continuous (quasi-continuous) work.*

Let's name the mechanical system consisting of two systems of mobile elements, making mirror symmetric repeating movement on a special trajectory, <u>the Force Element</u> - in abbreviated form **"FE"**.

FE can be united in groups for teamwork.

6.1. Group of FE with the phase shift equal to zero.

Let's admit, we have united in group n force elements.

At us the closed mechanical system consisting of a massive body (case) M_c and group of FE, total mass during the moment $t_0 = 0$, equal

$$M_{nc} = \sum_{1}^{n} M_{1c}$$

If the beginning of a running cycle at all FE entering into group, coincides, that is phase shift at all FE is equal to zero, for a running cycle all mechanical system will move on distance (according to (24) and (32)):

$$y_{\max} = \frac{k}{2\pi} \frac{(n\,M_{1c})\,R}{\left(M_c + (n\,M_{1c})\right)} \qquad \textbf{(42)}$$

, where: $k \approx 2.437653393$

After end of a running cycle each FE is necessary for resulting again in an initial condition, that is, to make the actions listed in the beginning of charter 5.

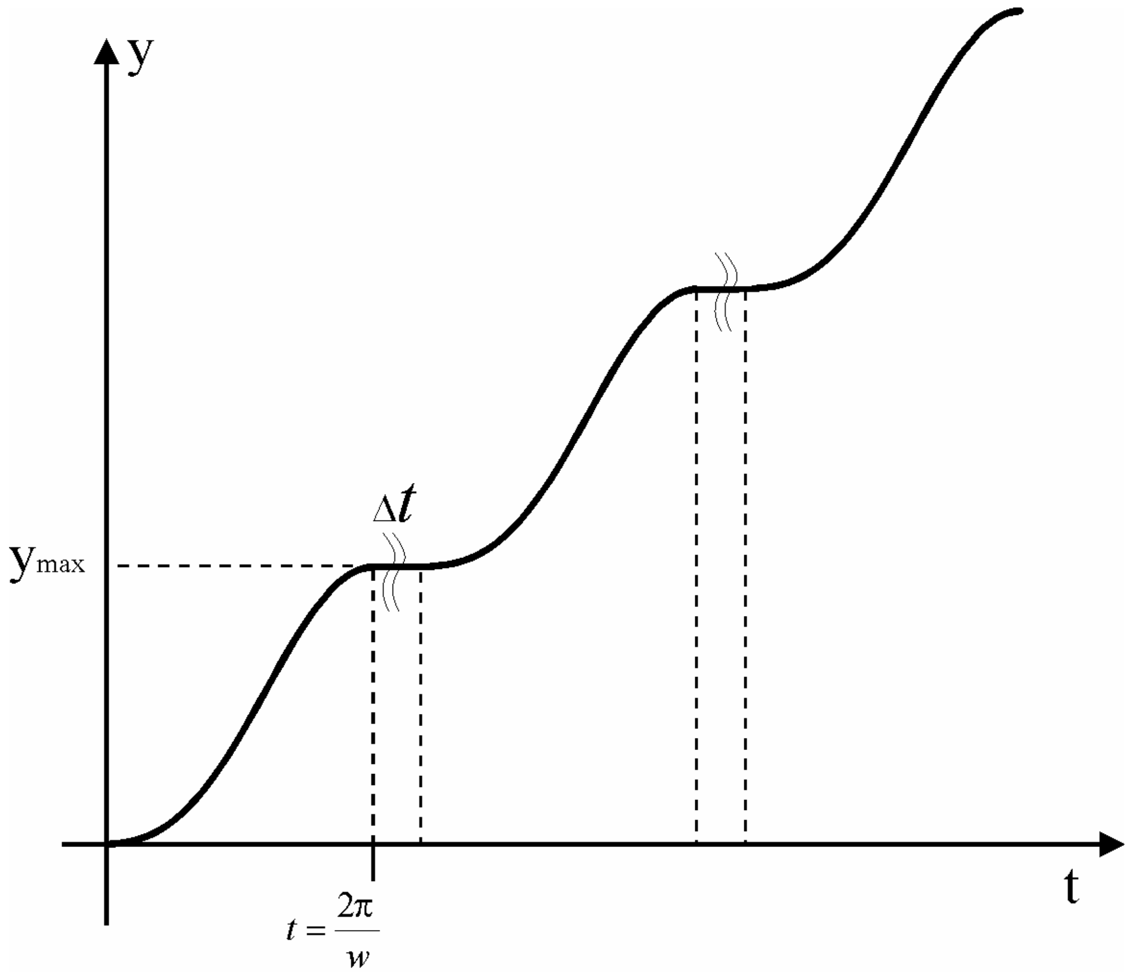

Fig. 16

The plot of moving of similar system is presented in a Fig. 16. Intervals of time Δt on the plot define time of restoration of group FE in an initial condition.

6.2. Group of FE with the phase shift which is distinct from zero.

It is possible to organize work of group of FE in such a manner that the beginning of a running cycle at each FE will differ from previous FE.

Let's admit, phase shift at each FE from group n elements will be equal

$$\Delta\alpha = \frac{2\pi}{n}$$

i.e., when first FE from group begins a running cycle, last FE finishes it.

In a Fig. 17 the conditional scheme of work of group of FE from 8 elements is presented.

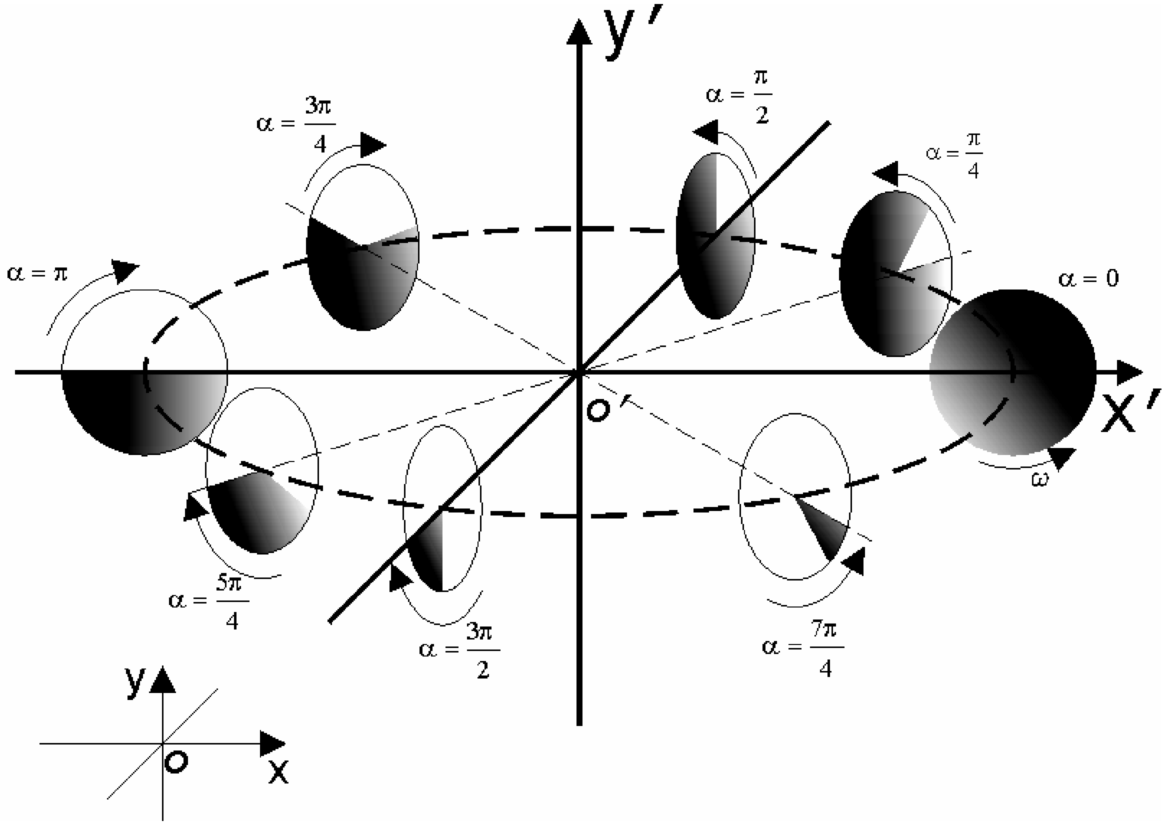

Fig. 17

Let's admit that each FE after end of a running cycle at once begins a new running cycle. Time of restoration of FE in a working condition it is passed for simplification.

Mass of the mobile elements participating in movement in system from n FE:

$$M_{1n} = \sum_{k=0}^{n-1} \left(\frac{M_{1c} \left(2\pi - \alpha - \frac{2k\pi}{n} \right)}{2\pi} \right), 0 \le \alpha \le 2\pi$$

At the big number n, i.e., accordingly, at small $\Delta\alpha$, it is possible to consider mass of the mobile elements participating in movement, a constant.

Average value of this mass: $M_{1n} = n\dfrac{M_{1c}}{2}$

, where M_{1c} - mass of one FE.

In a Fig. 18 the plot of change of masses of mobile elements of system from 4 FE is presented.

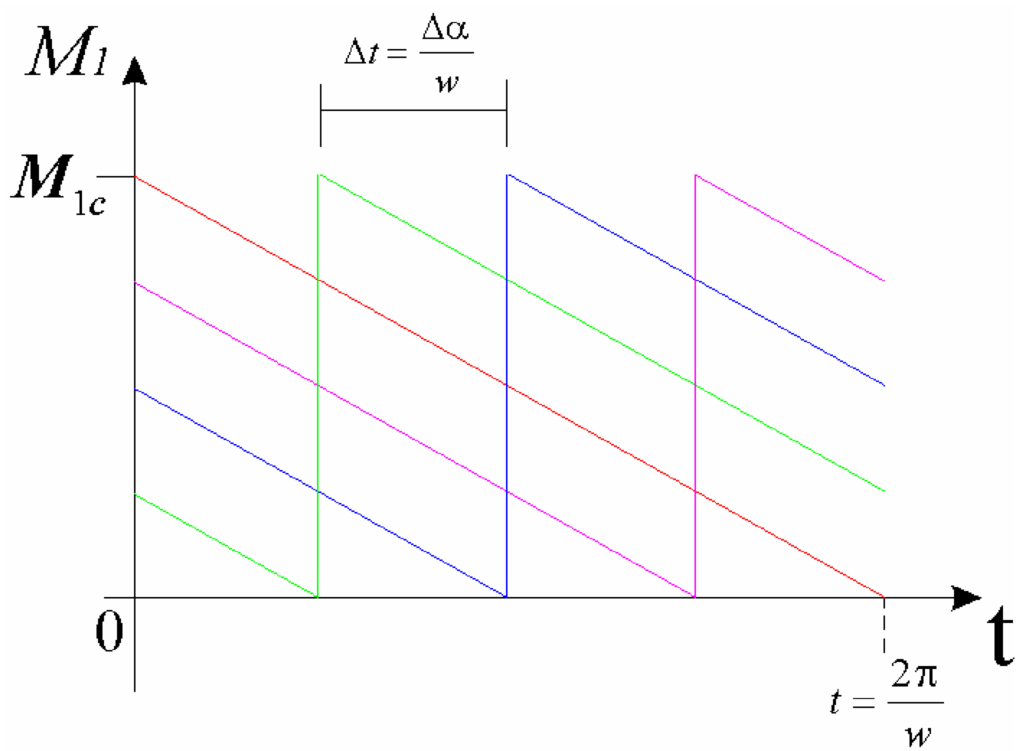

Fig. 18

The center of masses of each of FE moves on dependence (8).

$$y'_{1i} = \frac{R\,(1-\cos\ \ \alpha+i\,\frac{2\pi}{n}\ )}{2\pi-\ \ \alpha+i\,\frac{2\pi}{n}}\ ,0\le\alpha\le 2\pi$$

The schedule of moving of the centers of masses of mobile elements of system with 4 FE is presented in a Fig. 19.

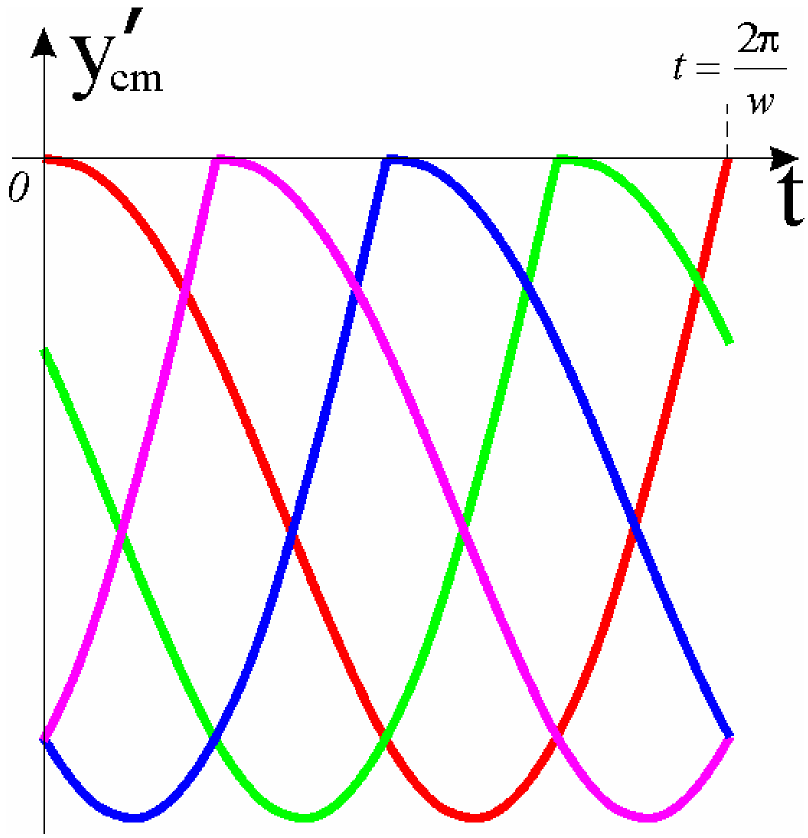

Fig. 19

The sum of quantities of movements of FE of the system of the coordinates connected with closed mechanical system is defined by expression:

$$Q_{1n} = \sum_{i=1}^{n} M_{1i} V_{1yi} \qquad \textbf{(43)}$$

In a Fig. 20 the schedule of change of momentum of FE system with phase shift is presented.

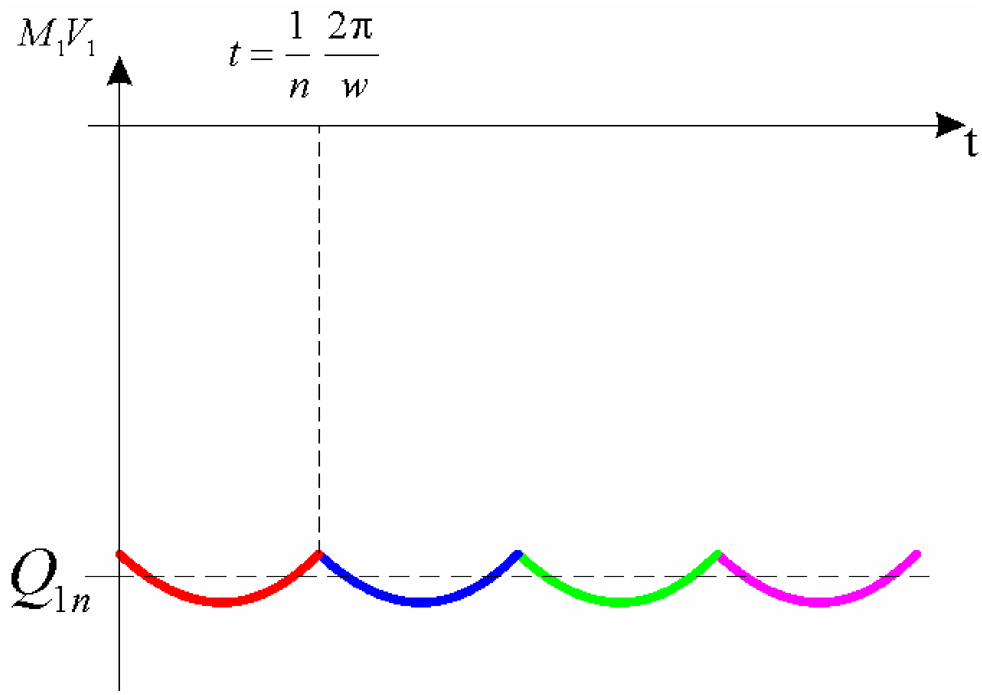

Fig. 20

*

Average value of a total momentum of FE in mobile coordinates system will be:

$$Q_{1n} = -\frac{1}{4}\frac{M_{1c}\ w\ R\,(\ln(2) + \ln(\pi) - \mathrm{Ci}(2\pi) + \gamma)\,n}{\pi^2} \tag{44}$$

From the law of preservation of a momentum of the closed mechanical system follows:

$$(M_c + n\ M_{1c})\left(\frac{d}{dt}\mathrm{y}_{0n}(t)\right) + Q_{1n} + M_{1n}\left(\frac{d}{dt}\mathrm{y}_{0n}(t)\right) = 0 \tag{45}$$

where:

$M_c + M_{1n}$ - mass of a part of mechanical system with which center of masses the reference mark of mobile coordinates system X'O'Y' is connected.

$\frac{d}{dt}y_{0n}(t)$ - speed of mobile coordinates system X'O'Y' in a projection to axis Y of motionless coordinates system XOY.

M_{1n} - mass of mobile elements of FE **(43)**.

Part of expression:

$Q_{1n} + M_{1n} \ \dfrac{d}{dt} y_{on}(t)$ - defines a momentum of FE in motionless coordinates system XOY.

The decision of the differential equation **(44)** with respect to $y_{on}(t)$, in view of entry conditions:

$y_{on}(0) = 0$ (initial speed of system is equal to zero),

Gives following result:

$$\boxed{y_{0n}(t) = \frac{1}{4} \frac{R\, w\, M_{1c}\, n\, (\ln(2) + \ln(\pi) - \mathrm{Ci}(2\pi) + \gamma)\, t}{\pi^2\, (M_c + M_{1c}\, n)}} \qquad \textbf{(46)}$$

The analysis of expression (46) shows, that the system with n FE, the beginning of a running cycle of each differs from the beginning of a running cycle of previous FE on size $\Delta t = \dfrac{1}{n}\dfrac{2\pi}{w}$, moves in regular intervals (Fig. 21). In a Fig. 21 red color represents a trajectory of movement of system from **<u>three</u>** FE. (at $n \longrightarrow \infty$, the curve $y_{0n}(t)$ in a Fig. 21 comes nearer to a straight line.)

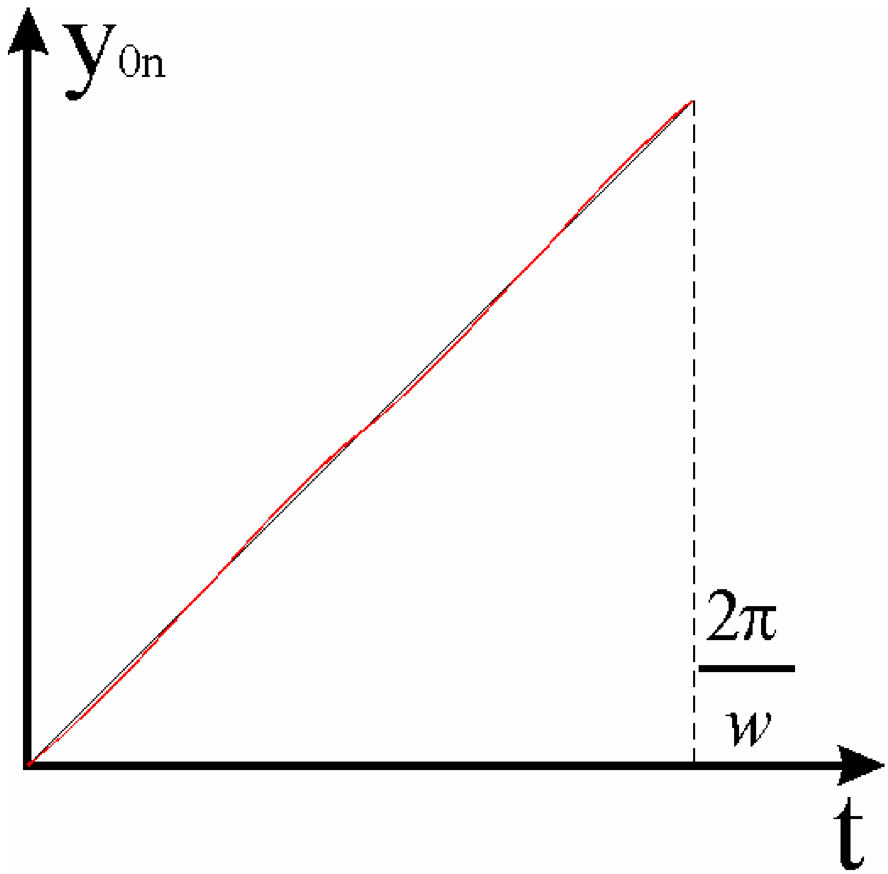

Fig. 21

Calculation is made of a condition, that each FE after end of a running cycle at once begins a new running cycle. Time of restoration of FE in a working condition is lowered for simplification.

The system moves in regular intervals only at $w \neq 0$.

Speed of all closed system is a derivative on time from the coordinate defining the center of masses of system:

$$V_{y0n} = \frac{d}{dt} y_{0n}(t)$$

$$\boxed{V_{y0n}(t) = \frac{1}{4} \frac{R\, w\, M_{1c}\ n\ (\ln(2) + \ln(\pi) - \mathrm{Ci}(2\pi) + \gamma)}{\pi^2\ (M_c + M_{1c}\ n\ )}} \tag{47}$$

Acceleration of system as a derivative on time from speed, **it is equal to zero**:

$$a_{0n} = \frac{d}{dt} V_{y0n} = 0$$

That is, at **any moment the system moves without acceleration**.

It is possible to calculate force from which the system of FE is capable to move all mechanical system.

The force developed by one FE is certain in (38):

For group of FE with phase shift total force:

$$F_{1n} = \sum_{k=0}^{n-1} \mathrm{F}_1 \left(\alpha + \frac{2\,k\,\pi}{n} \right)$$

At a plenty of FE in group, it is possible to accept average value of total force:

$$\boxed{F_{cp} = \left(\pi I + \ln(\pi) + \ln(2) - Ci(-2\pi) + \gamma\right) \frac{w^2 R}{4\pi^2}\, n\, M_{1c}} \tag{48}$$

$$F_{cp} \approx 2.43765 \times \frac{w^2 R}{4\pi^2}\, n\, M_{1c}$$

By analogy to gyroscopes at which the gyroscopic moment is result Coriolis forces of inertia, at FE **force of moving is result of centripetal forces of inertia**.

At continuous group work each FE changes the kinetic energy (39)**.**

For a time unit, quantity of the "lost" energy it is equal to quantity of the energy transferred by FE for restoration. At big n and continuous restoration of FE, it is possible to consider work on restoration of FE uniform.

The capacity necessary for restoration of FE in a working condition:

$$N = \frac{1}{8\pi} n M_{1c} w^3 R^2 \tag{49}$$

The note:

Value of capacity is found from a condition, that elements of mobile systems of FE completely stop after end of a running cycle. In real devices it to do it is inexpedient. It is recommended to change speed of mobile elements only on one coordinate.

In that case, it is possible to use a stock of kinetic energy of mobile system for translation of FE in a working condition. Thus value (49) will be much less.

7. *Force elements with the increasing mass of mobile elements.*

The task represented in a Fig. 1 can be a little bit modified.

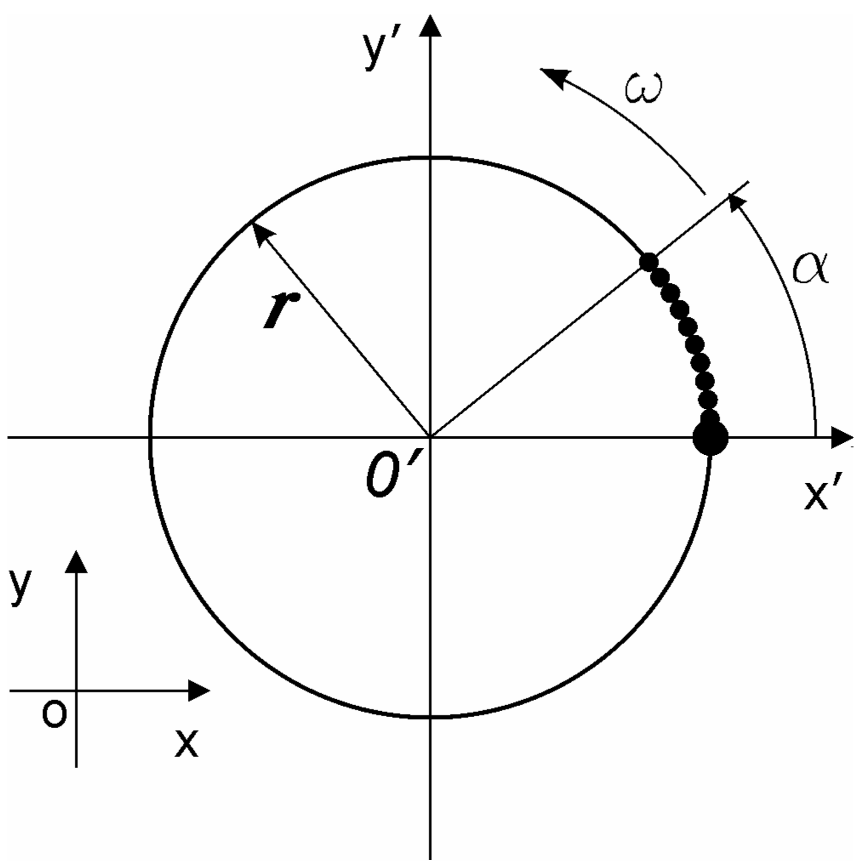

Fig. 22

Around of the center of masses of a massive body M_c, on a circle of radius $\boldsymbol{R}$ the system of mobile elements moves with constant speed. Elements are put forward from some "source" in total mass M_{1c} (during the initial moment of time), connected with a body M_c. The system of bodies is allocated **continuously** and **in regular intervals**. Through time $\Delta \boldsymbol{t} = \frac{2\pi}{\boldsymbol{w}}$ after the beginning of promotion of elements, i.e. after end of a running cycle, all trajectory of movement of mobile elements will be filled by these elements.

To repeat a running cycle, **not breaking position of the center of masses of all system**, it is necessary simultaneously and to move mobile elements to a point from which elements are put forward on a working trajectory in regular intervals.

For the working period the closed mechanical system represented in a Fig. 22 will move the same as also system in a Fig. 1, on distance

$$y_{c\max} = \frac{k}{2\pi} \frac{M_{1c}\ R}{\left(M_c + M_{1c}\right)}\text{, where}$$

$$k = \left(-Ci(2\pi) + \gamma + \ln(2) + \ln(\pi)\right) \approx 2.437663393$$

$$\boxed{y_{c\max} = 0.38796 \frac{M_{1c}}{\left(M_c + M_{1c}\right)} R}$$ (see the formula (23), (24))

Corresponding drawings of moving of the center of masses of the case of system and the center of masses of all system as a whole are resulted in Fig. 23 and Fig. 24

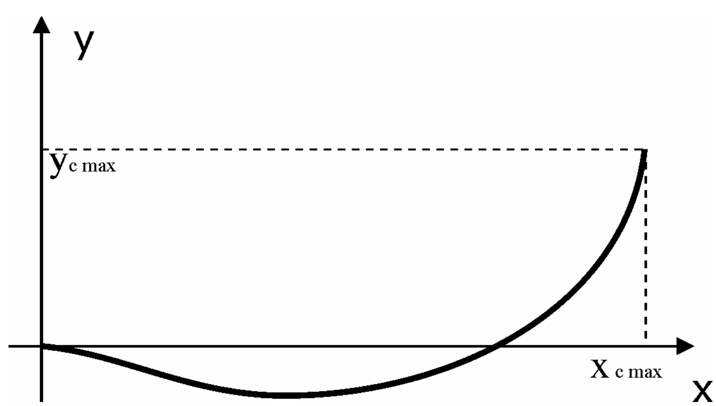

Fig. 23

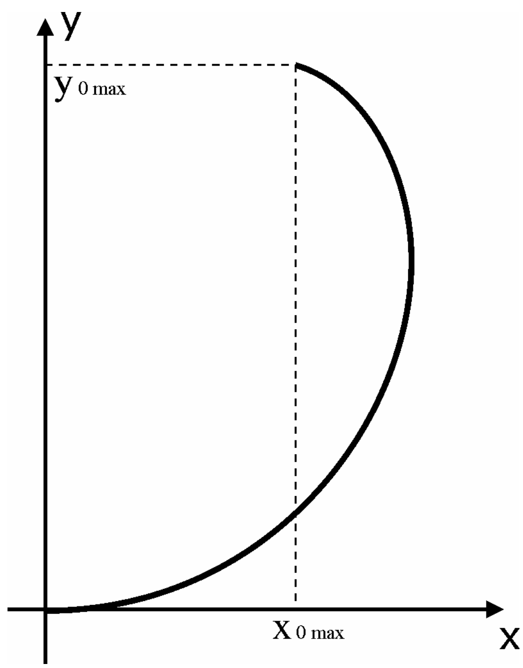

Fig. 24

8. *Materials for systems of mobile elements.*

As mobile elements in FE various materials of various phase conditions - solid, liquid, gaseous can be used.

The organization of a trajectory of moving of mobile elements can be also various, proceeding from properties of substance of mobile elements.

9. *Restoration of force elements in a working condition.*

In a Fig. 25 the scheme of restoration of FE is presented to a working condition. In figure:

a - b) - the Running cycle. Mobile elements gather in a final point of a trajectory;

c) - Moving the collected mobile elements to the center of a trajectory;

d -e) - Accommodation of mobile elements on a working trajectory and giving of speed by it.

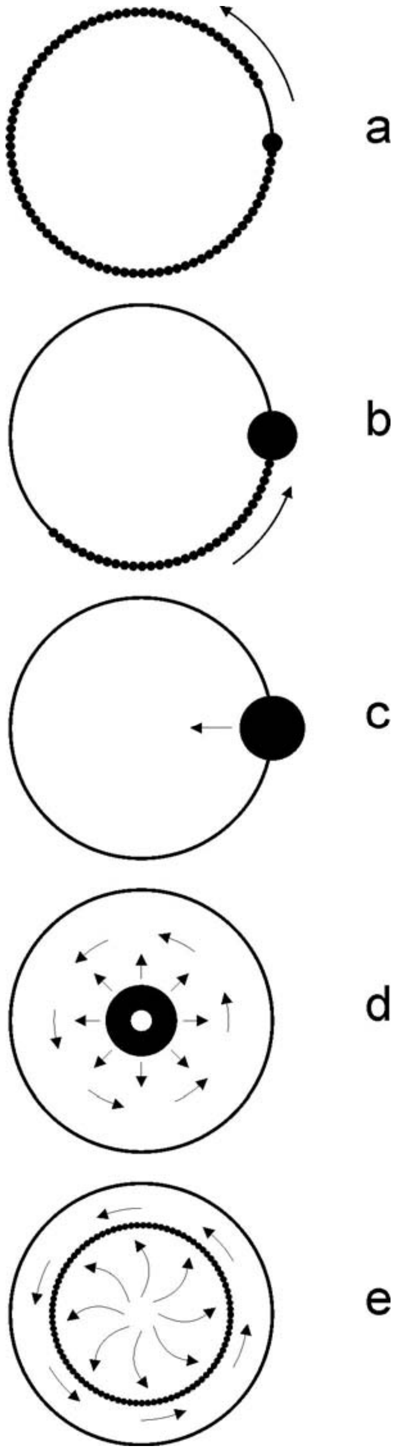

Fig. 25

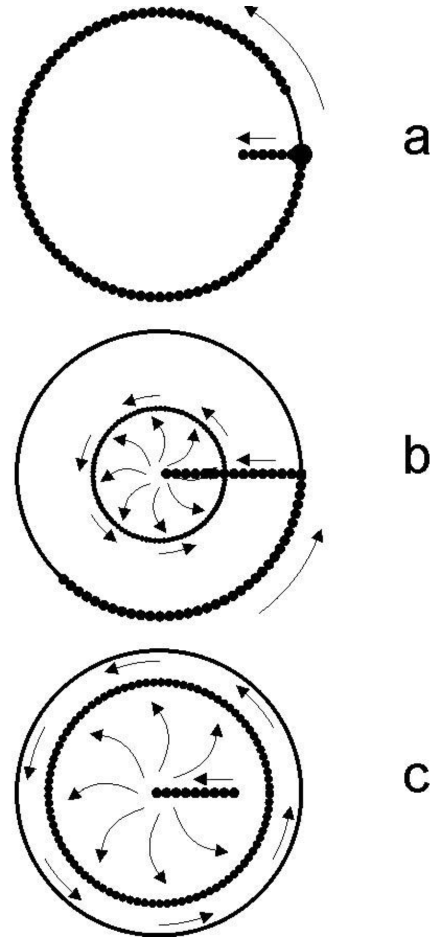

Fig. 26

For reduction of time of restoration of FE and for preservation of a part of kinetic energy of mobile elements, it is possible to make a conclusion of mobile elements to a working trajectory during a running cycle (Fig. 26). In that case, practically at once after end of a running cycle it is possible to begin a new running cycle.

In a Fig. 25 and the Fig. 26 are presented variants of restoration of FE with reduction of mass of mobile elements. For FE with increase in mass of mobile elements (Fig. 22) principles of restoration practically same, only varies a direction of gathering and dispersal of mobile elements.

There is an opportunity of association in one force block of FE with increase (↑) and reduction (↓) masses of mobile elements (Fig. 27).

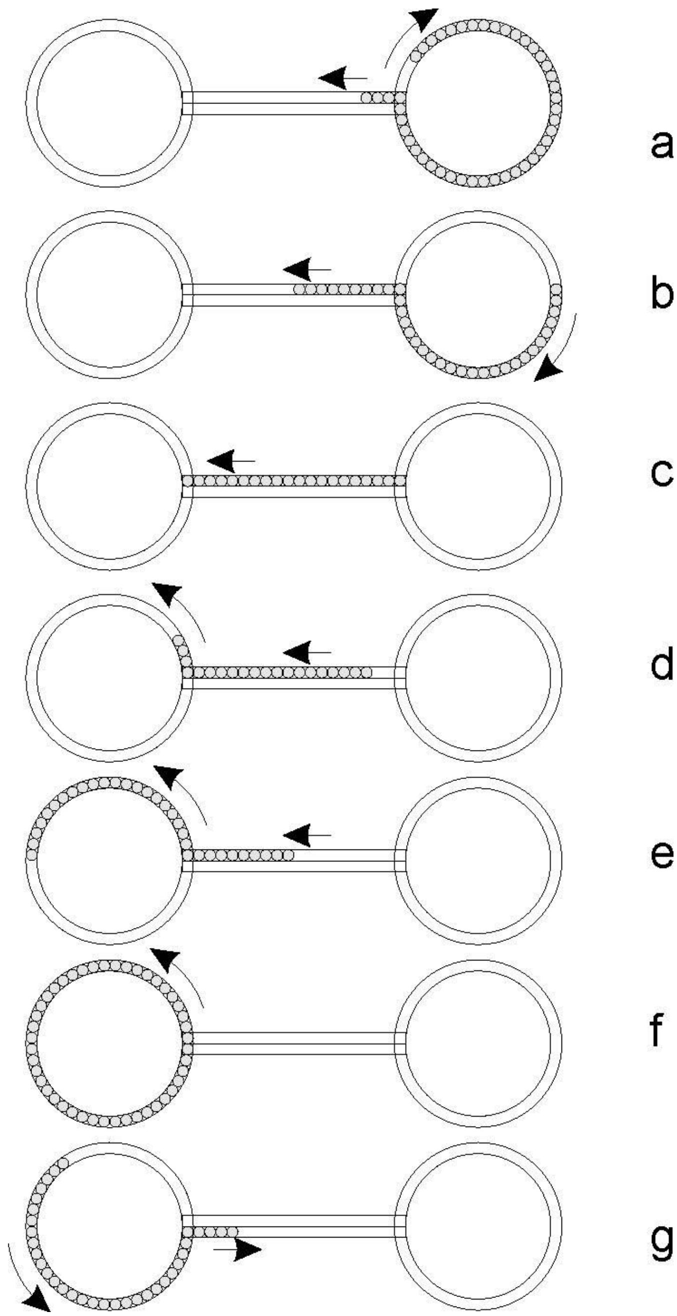

Fig. 27

In a Fig. 27:

a - b) - the right part of FE (on fig.) works, as FE $\downarrow$;

c) - transition of mobile elements;

d-e-f) - the left part of FE works, as FE $\uparrow$;

f -g) - the left part of FE works, as FE $\downarrow$, transition to a).

It is necessary to make a reservation at once, that such association is possible only: or for single FE, or for group of FE with zero phase shift. As a total momentum of mobile elements in FE $\uparrow$ and in FE $\downarrow$, if their phases it are equal among themselves, equal to zero.

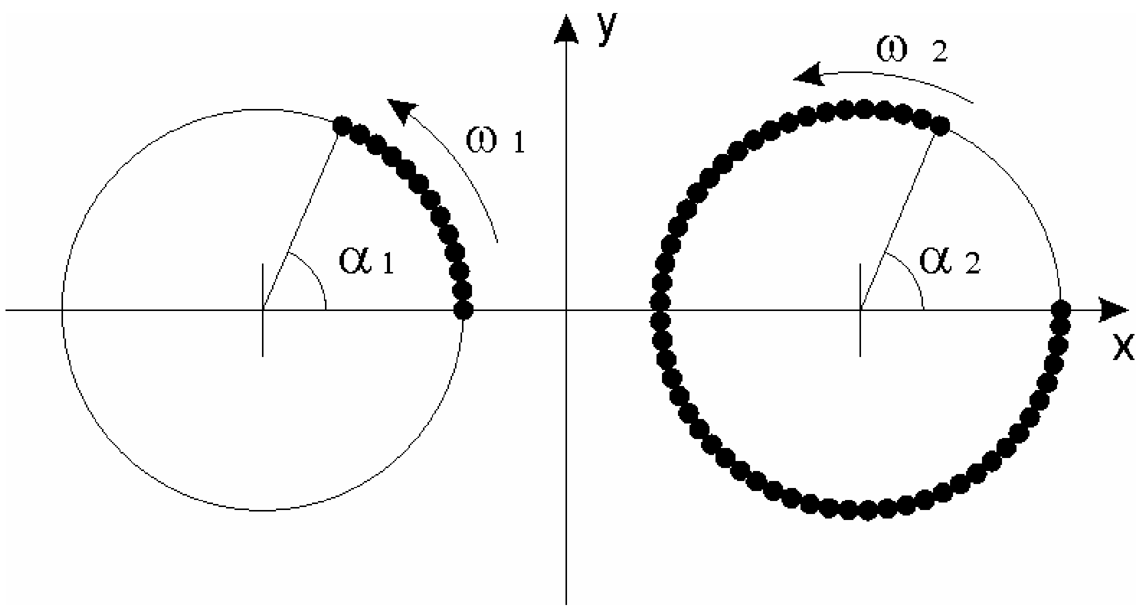

Fig. 28

In a Fig. 28 integration FE $\uparrow$ and FE $\downarrow$ with an identical phase is presented.

At $w_1 = w_2$ and $\alpha_1 = \alpha_2$ a momentum of two systems of mobile elements to equally momentum of one, moving on a circle, in regular intervals allocated mass.

$$Q_{\alpha 1\uparrow} + Q_{\alpha 2\downarrow} = Q_{\alpha = 2\pi} = 0$$

10.*Practical use.*

Force elements, separately or incorporated in groups, can be used as engines of any devices for moving to any conditions and environments.

The material system moving by means of FE, will possess very high maneuverability as the kinetic condition of system during movement doesn't change.

Example.

The material system a lump of 1000 kg includes 10 FE with phase shift.

On a share of all FE 10 % from a lump of system are allocated.

The radius of a trajectory of FE is equal 1 m.

To define the minimal kinematic characteristics of FE to tear off all material system from a surface of the Earth.

Solution.

Mass of one FE:

$$M_{1c} = \frac{1}{10} \times 0.1 \times M_0 = 10\ kg$$

Weight of all system at a surface of the Earth:

$$P_0 = M_0 g \approx 9800\ N$$

From (48):

$$w = 2\pi \sqrt{\frac{F_{cp}}{2.43766 \times n\ M_{1c} R}}$$

$$w = 2\pi \sqrt{\frac{9800}{2.43766 \times 10 \times 10 \times 0.5}} \approx 56\ \frac{radian}{s}$$

$$w = 56\ \frac{radian}{s} \approx 9\ s^{-1} \approx 550\ min^{-1}$$

The power necessary for continuous work of all FE, is defined from (49)

$$N = \frac{1}{8\pi} n M_{1c} w^3 R^2$$

$$N = \frac{1}{8\pi} \times 10 \times 10 \times 56^2 \times 0.5^2 \approx 170\,000\ \text{Watt} = 170\ \text{kW}$$

This maximal value of power. In real devices it should be much less. (See the note to the formula (49)).

Speed of system at absence of gravities is defined from (47):

$$V_0 \approx 0.17 \frac{m}{s}$$

<u>Note</u>.

Calculations in the given example are made very much approximately as real dynamic both geometrical characteristics of FE and methods of restoration of FE in a working condition aren't considered.

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